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**Pioneering CFD Software for Education & Industry**

**CHAM Case Study – Non-Newtonian Flow**  
PHOENICS applied to Axisymmetric Flow of a Pseudo plastic Fluid in  
Annual Passages

The case considered is the steady, laminar, isothermal, axisymmetric flow of a pseudoplastic fluid in an annulus. The shear-thinning behaviour of the non-Newtonian viscous fluid is described by the power-law model. The main input parameters specified by the client are:

- Outer diameter  $D_i = 0.065\text{m}$  Inner
- diameter  $D_o = 0.048\text{m}$ .
- Volumetric flow rate  $Q = 3000 \text{ l/hr}$
- Apparent dynamic viscosity,  $\mu = K\dot{\gamma}^{(n-1)}$  where the consistency  $K=11.09 \text{ Pa}\cdot\text{s}^n$ , the power-law index  $n=0.265$ , and  $\dot{\gamma}$  is the mean rate of strain.

The pipe length has been chosen arbitrarily as  $0.25\text{m}$ , which corresponds to a length  $L \approx 15d$ . Here,  $d$  is the hydraulic diameter, which is given by  $d=D_o-D_i=0.017\text{m}$ .

The fluid density  $\rho$  was not specified by the client, and so in the present computations, the density of water has been presumed, i.e.  $\rho=1000 \text{ kg/m}^3$ . The working fluid is believed to be some sort of fluid food, and so the density of water is likely to be a good first approximation. The value of the power-law parameters  $K$  and  $n$ , correspond to a temperature of  $50.6^\circ\text{C}$ , as specified by the Client.

The inlet velocity  $w=Q/A=0.5523 \text{ m/s}$ , where  $Q=8.3333 \cdot 10^{-4} \text{ m}^3/\text{s}$  and the flow area  $A=\pi(D_o+D_i)(D_oD_i)/4 = 1.50875 \cdot 10^{-3} \text{ m}^2$ . The flow regime can be determined from the Generalised Reynolds number,  $Re^*$ , which is given by:

$$Re^* = \frac{\rho w d}{\mu_e} \quad (1)$$

where the effective viscosity  $\mu_e$  is given by:

$$\mu_e = K \left( b + \frac{a}{n} \right)^n \left( \frac{8w}{d} \right)^{n-1} \quad (2)$$



For circular pipe flow, the values of constants a and b in equation (2) are given as a=0.25 and b=0.75; whereas for flow in concentric annuli, these values depend on the value of  $\kappa = D_i/D_o$ . For the present case, with  $\kappa=0.7385$ , a=0.4986 and b=0.999. Therefore, equations (2) and (1) yield  $\mu_e=0.24649$  Pa.s and  $Re^*=38.09$ , which corresponds to laminar flow of the shear-thinning fluid.

Two computations are made with PHOENICS-2009. The first considers purely annular flow, whereas the second considers flow in an annulus with a double-cone obstruction located on the inner surface. The results of these computations are discussed very briefly in the following two sections.

### Purely Annular Flow

The PHOENICS CFD computation is made on an axisymmetric, cylindrical-polar mesh of 50 radial by 100 axial cells. No attempt is made to assess the mesh-sensitivity of the solution. The computation converges in less than a minute on a Dell Precision T7400 Intel Xeon 2.5GHz pc with 16GB RAM.

The predicted pressure drop is  $\Delta p=3.848$  kPa, which is within 2% of the analytical value. The analytic pressure drop of the power-law fluid is given by:

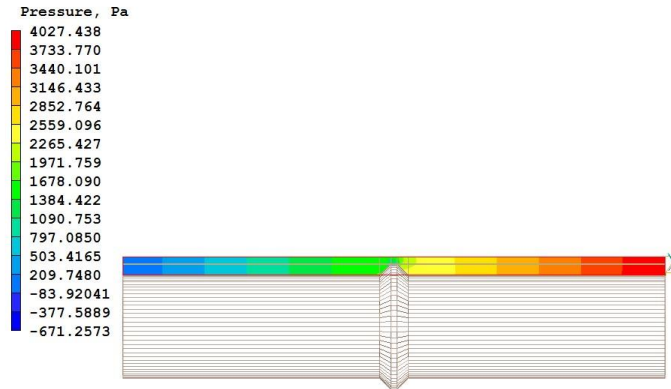
$$\Delta p = \frac{1}{2} \rho v^2 f \frac{L}{D} \quad (3)$$

where the friction factor  $f=64/Re^*$ . For the present case, the expected pressure drop is  $\Delta p=3.769$  kPa.

### Flow in an Annulus with a Double-Cone Obstruction

PHOENICS computations are made for this case on an axisymmetric, cylindrical-polar mesh of 61 radial by 165 axial cells. This calculation converges in less than 2 minutes on the Dell pc. Again, no attempt is made to assess the mesh-sensitivity of the solution. The double-cone geometry is represented by means of PARSOL, the cut-cell algorithm in PHOENICS. PARSOL captures complex geometries automatically on a background polar mesh by using cut cells at the fluid-solid interface. These cells are partially filled with solid and fluid.

The flow geometry and predicted pressure drop for this case are shown in Figure 1 below:

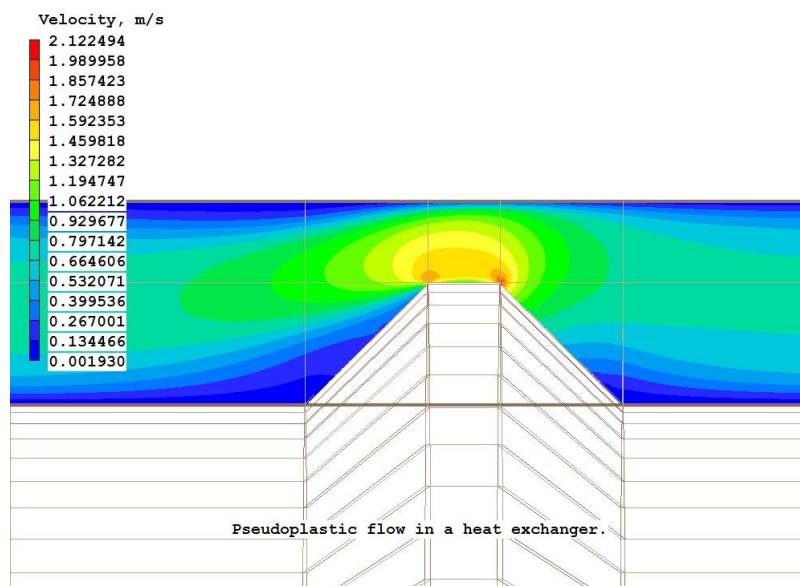


Pseudoplastic flow in a heat exchanger.

**Figure 1: Annulus with Double-Cone Obstruction: Predicted Pressure Drop.**

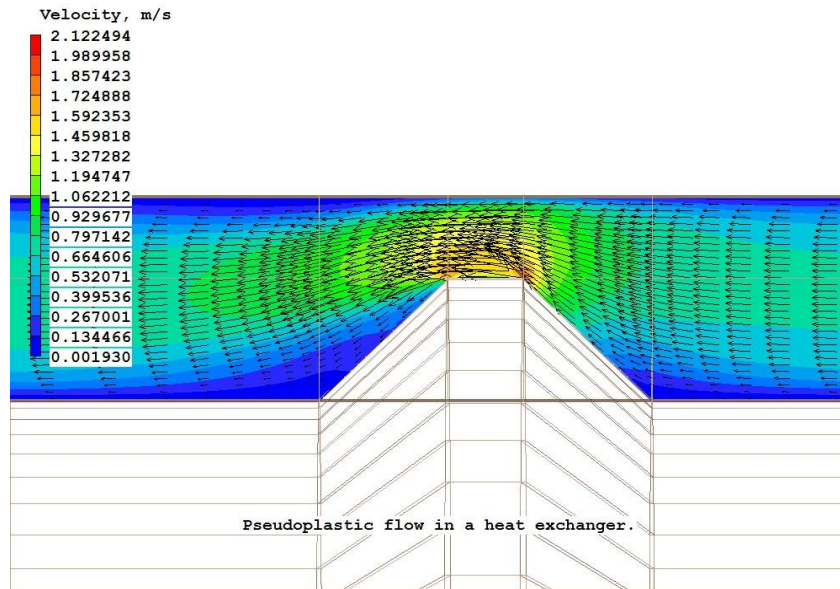
As can be seen from the figure, the predicted pressure drop is about 4KPa, which represents a 4% increase over that found for flow in a simple annulus.

The computed absolute velocity contours and velocity vectors are shown in Figures 2 and 3, respectively. For clarity, the velocity vectors are plotted on a mesh actually coarser than that used in the computations.



Pseudoplastic flow in a heat exchanger.

**Figure 2: Double-Cone Obstruction: Predicted Absolute Velocity Contours.**



**Figure 3: Double-Cone Obstruction: Predicted Velocity Vectors.**

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