

PHOENICS Predictions of Large Amplitude Internal Waves in the Ocean

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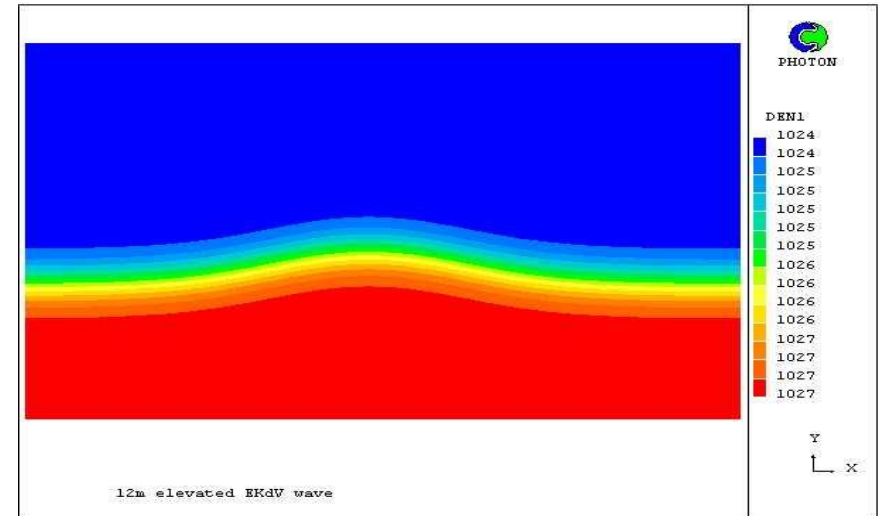
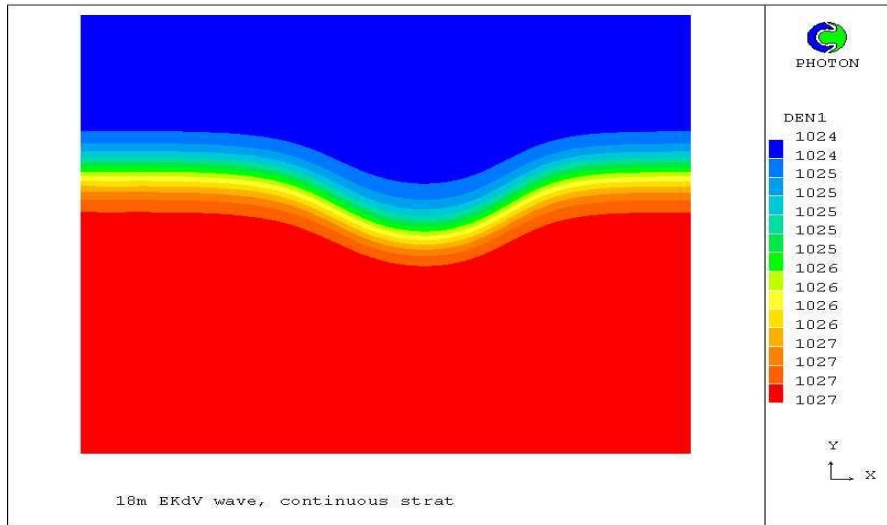
- ⌘ Large amplitude Internal waves in the ocean
- ⌘ Motivation
- ⌘ Mathematical formulation
- ⌘ PHOENICS case studies
- ⌘ Conclusions

Large amplitude internal waves in the ocean

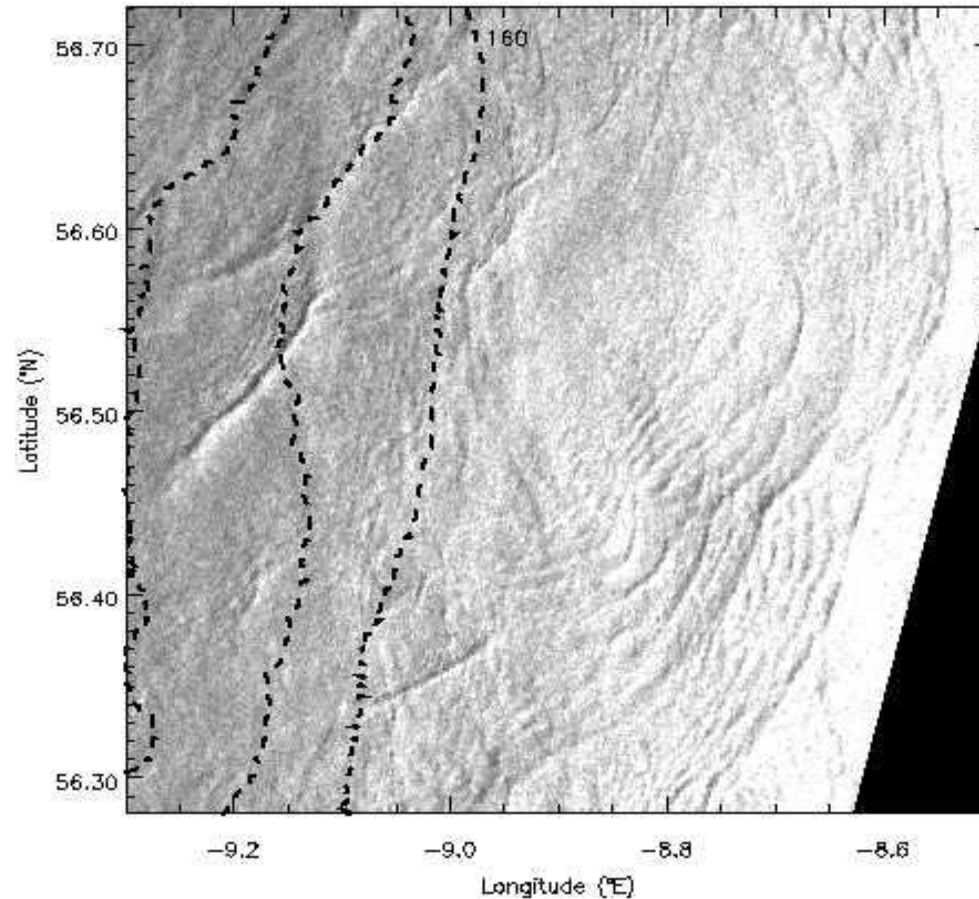
Large amplitude internal waves

⌘ Large amplitude internal waves

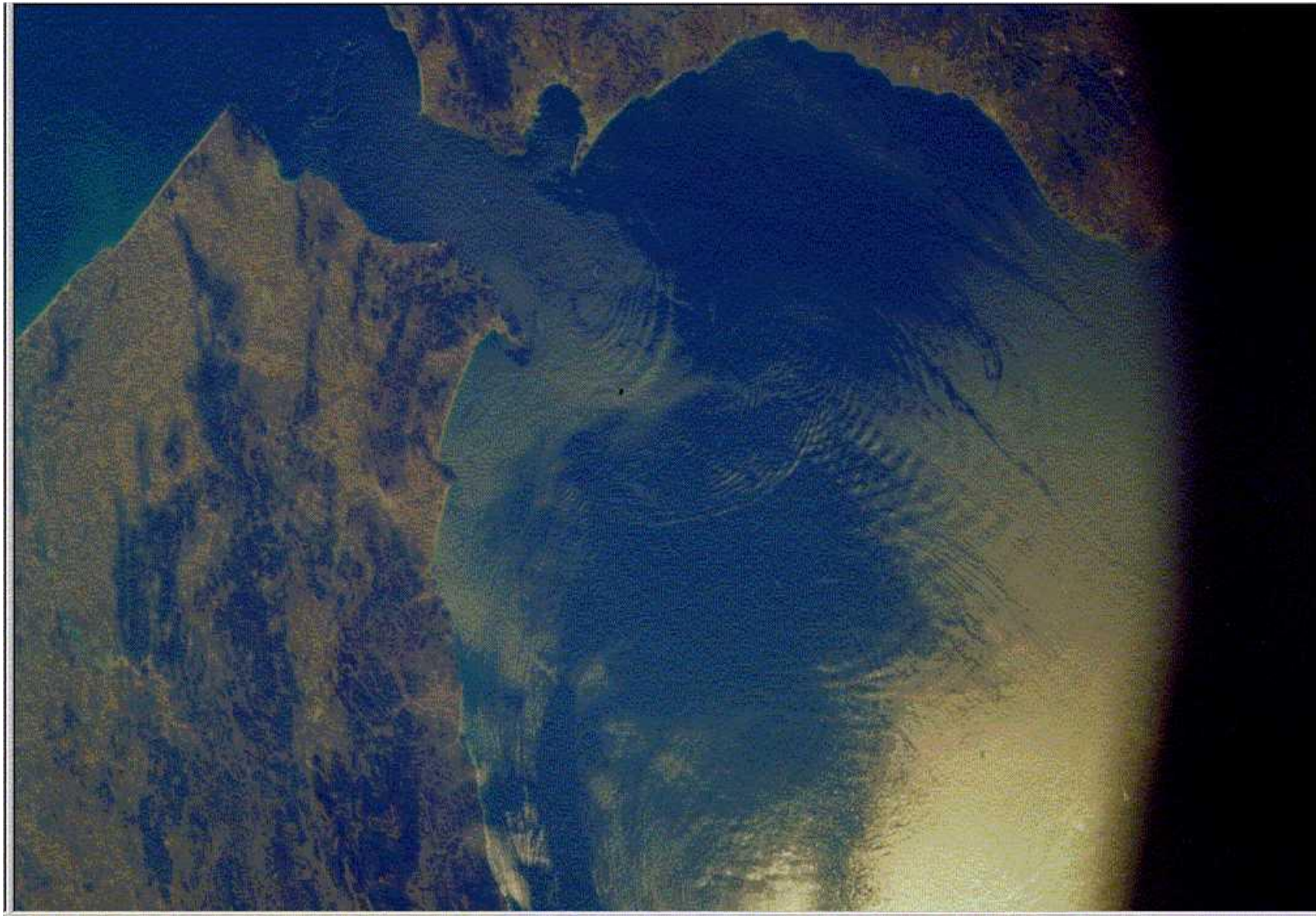
- Prevalent where stratified ocean is forced over bathymetry
 - Shelf edge regions (eg UK shelf)
 - Straits (eg Gibraltar)



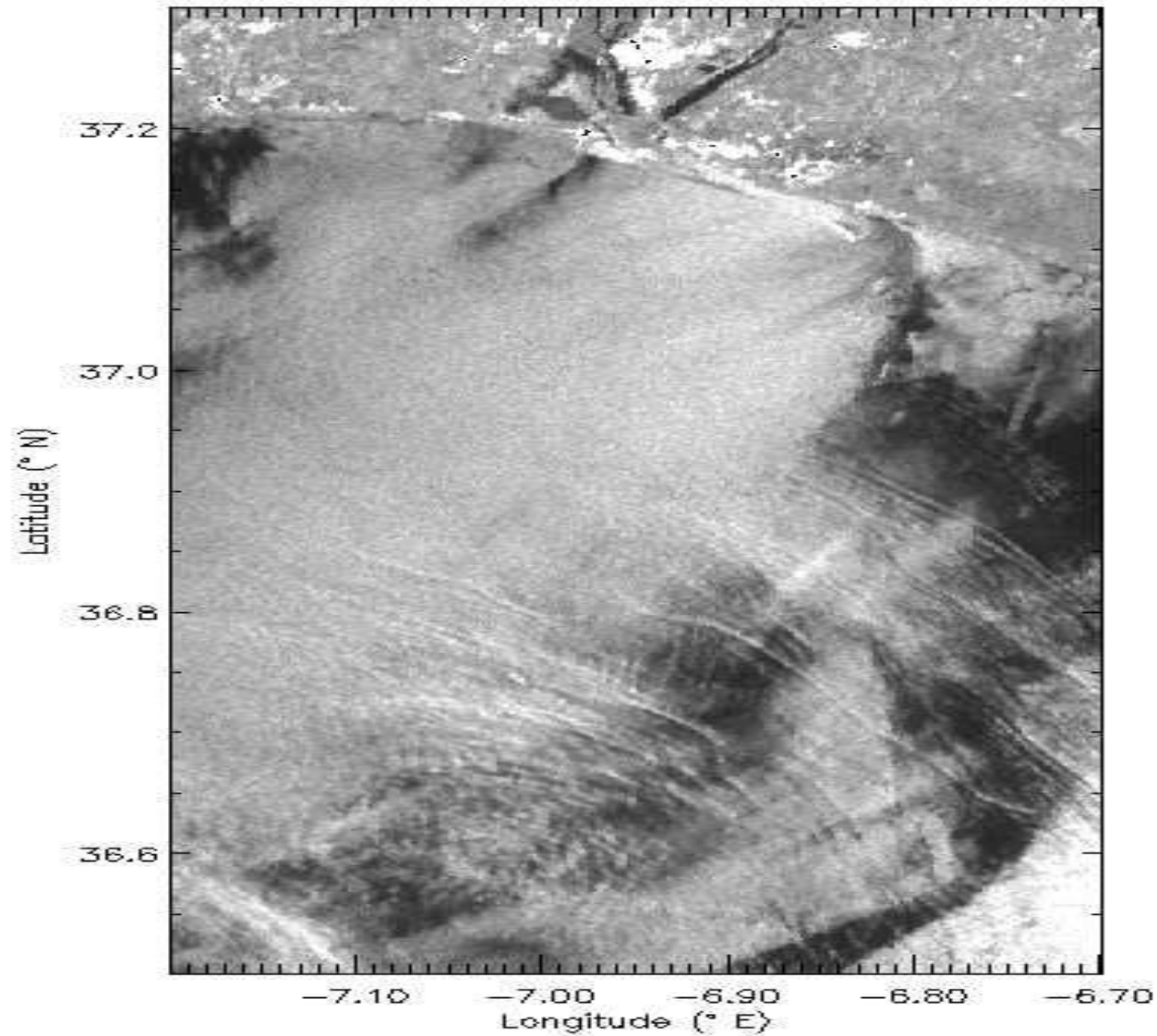
ERS-1 Synthetic Aperture Radar image of the Malin shelf-edge, 20th August 1995



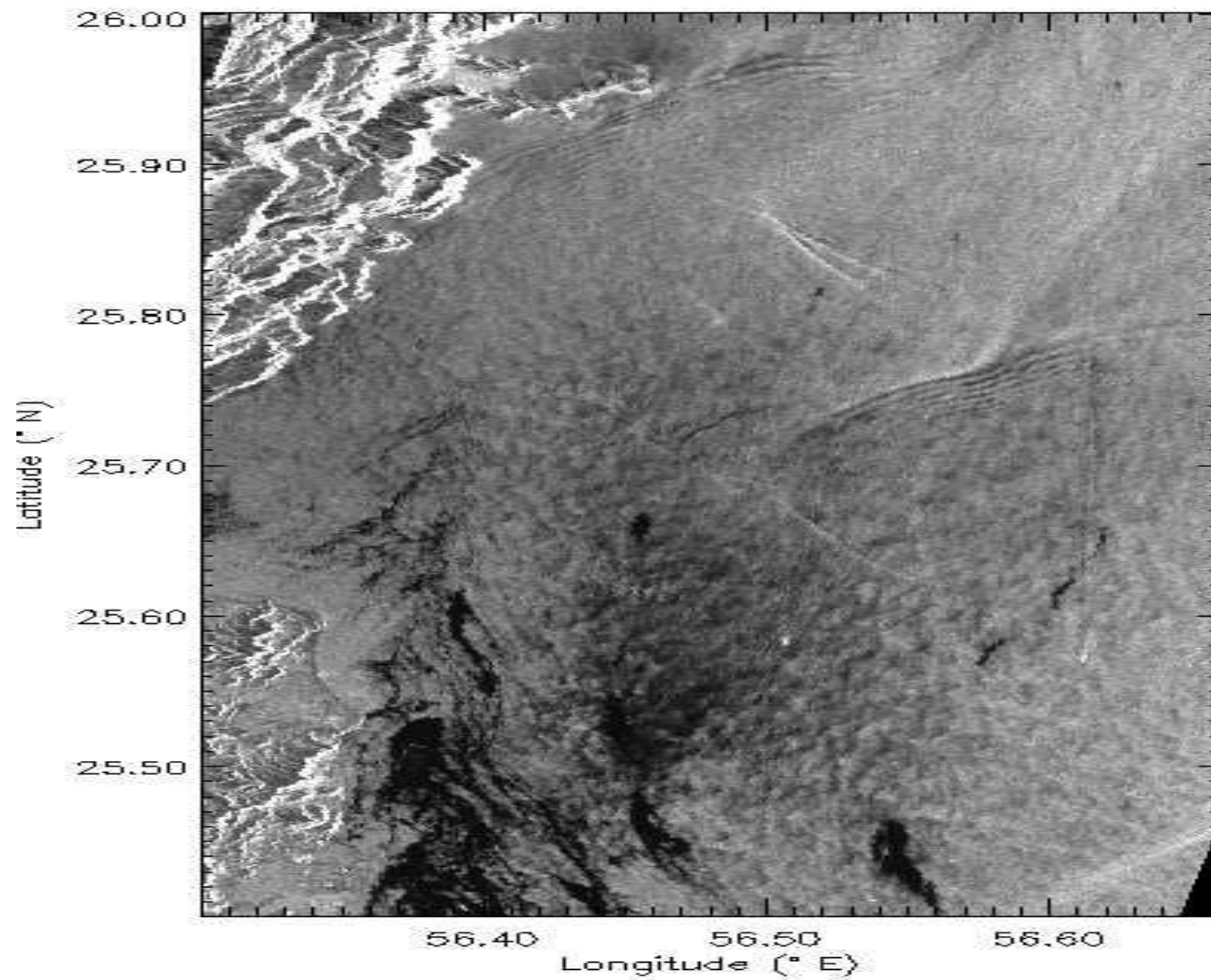
Space Shuttle Straits of Gibraltar 1989



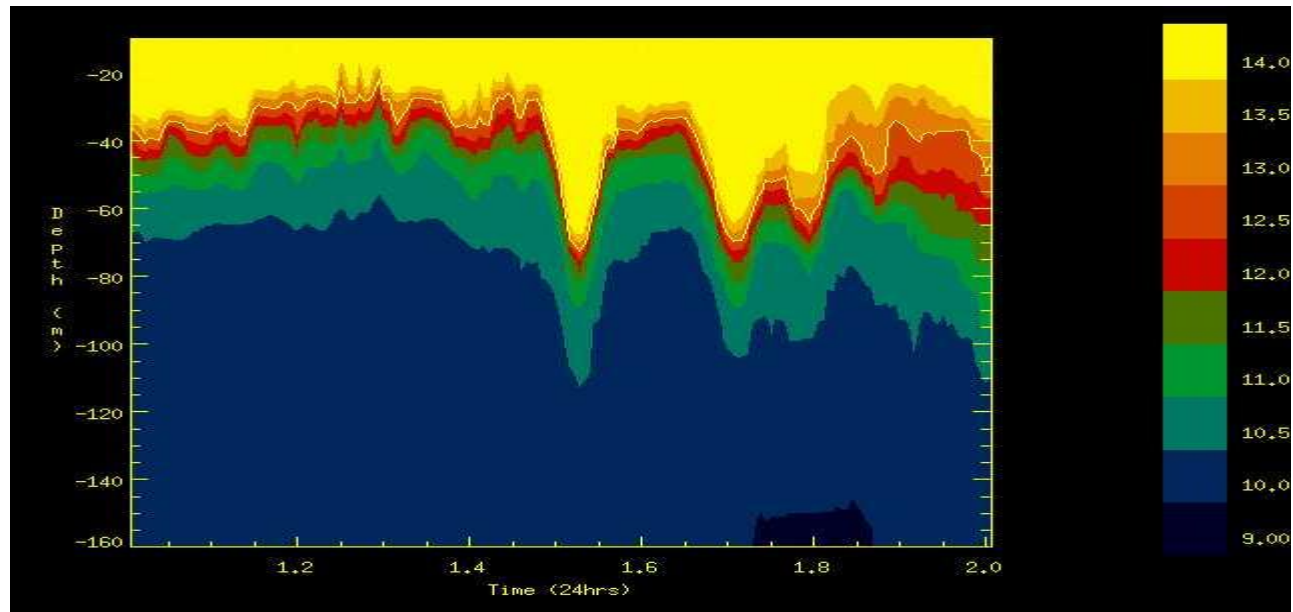
ERS-2 SAR image of Gulf of Cadiz July 1998



ERS-1 SAR image of Gulf of Oman Sept 1992



DERA thermistor chain/SAR image Malin Shelf 1995



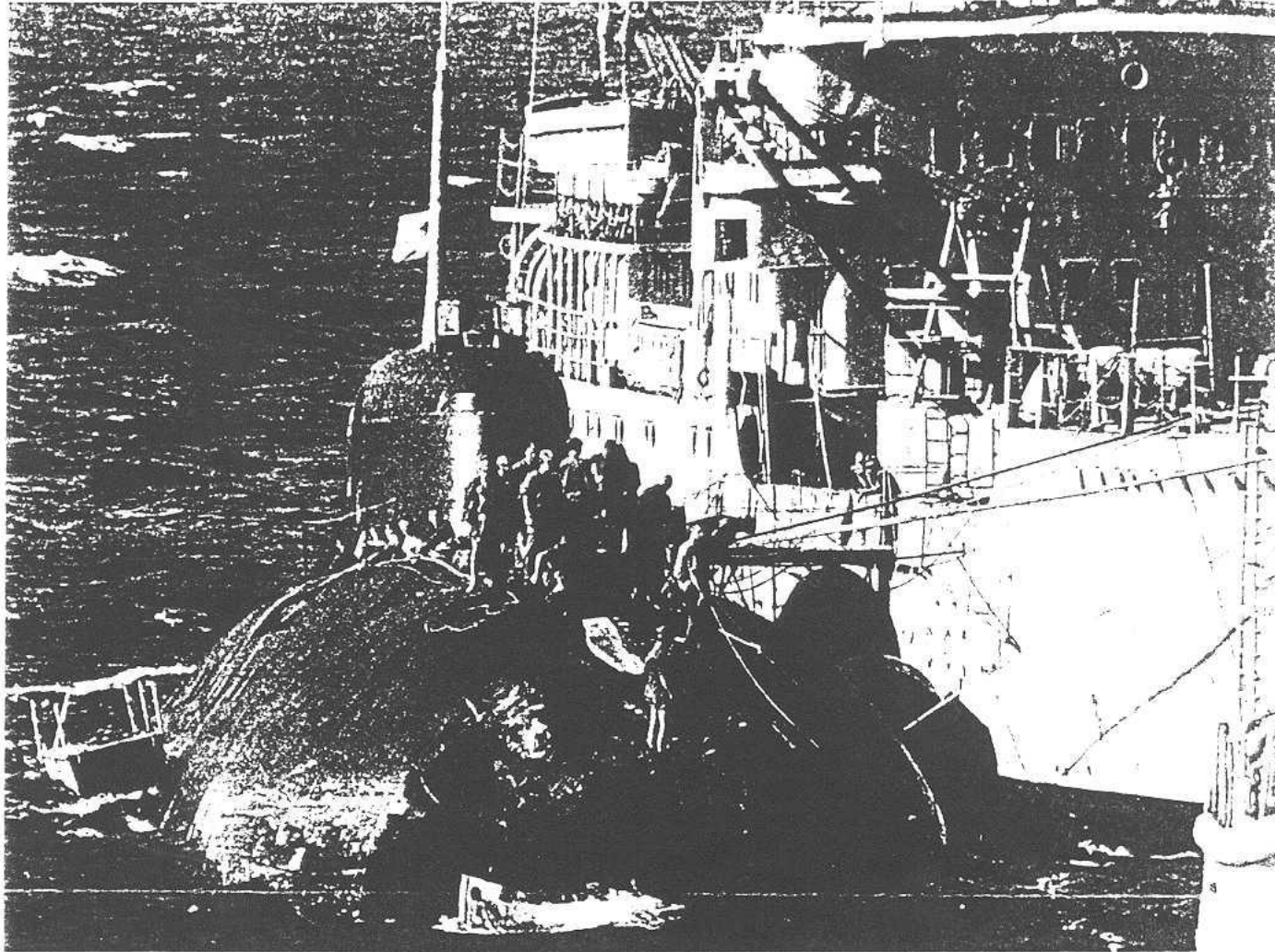
Motivation

Motivation

⌘ Large amplitude internal waves affect:-

- Stability of submersibles and moored oil platforms
- Distribution of nutrients and pollutants
- Acoustic propagation

Soviet Victor II SSN Straits of Gibraltar 1984



The extensive Soviet submarine operations have been accompanied by several casualties—the loss of four and possibly five nuclear submarines, and probably several diesel-electric units. This Victor II SSN is shown alongside a repair ship after colliding with a merchant ship while making a submerged transit of the Strait of Gibraltar. (Royal Navy)

Moored oil rig Andaman Sea October 1997



8.10.97 - ANDAMAN SEA - "STENA CLYDE" LISTING 3° DUE TO SOLITON -
NOTE CURRENT SLICK.

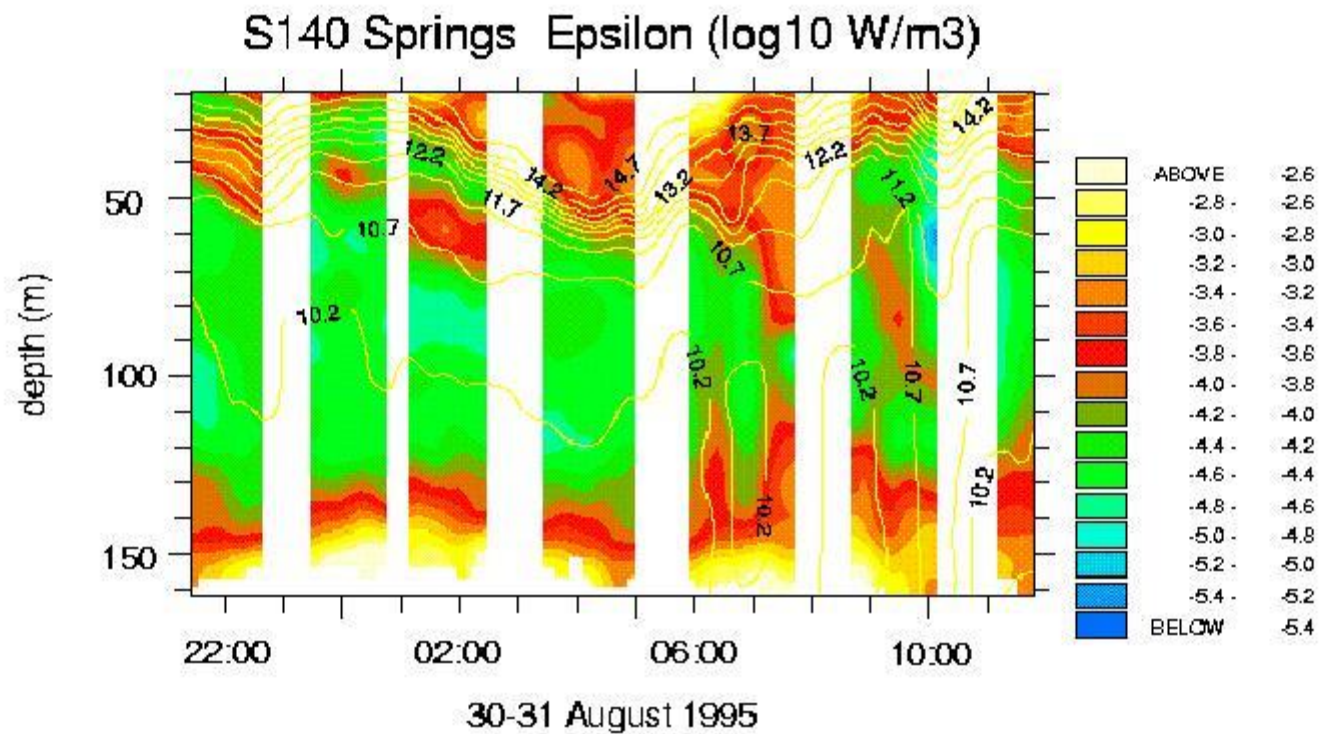
Moored oil rig Andaman Sea October 1997



18.10.97 - ANDAMAN SEA - "STENA CLYDE" LISTING 3° DUE TO SOLITON

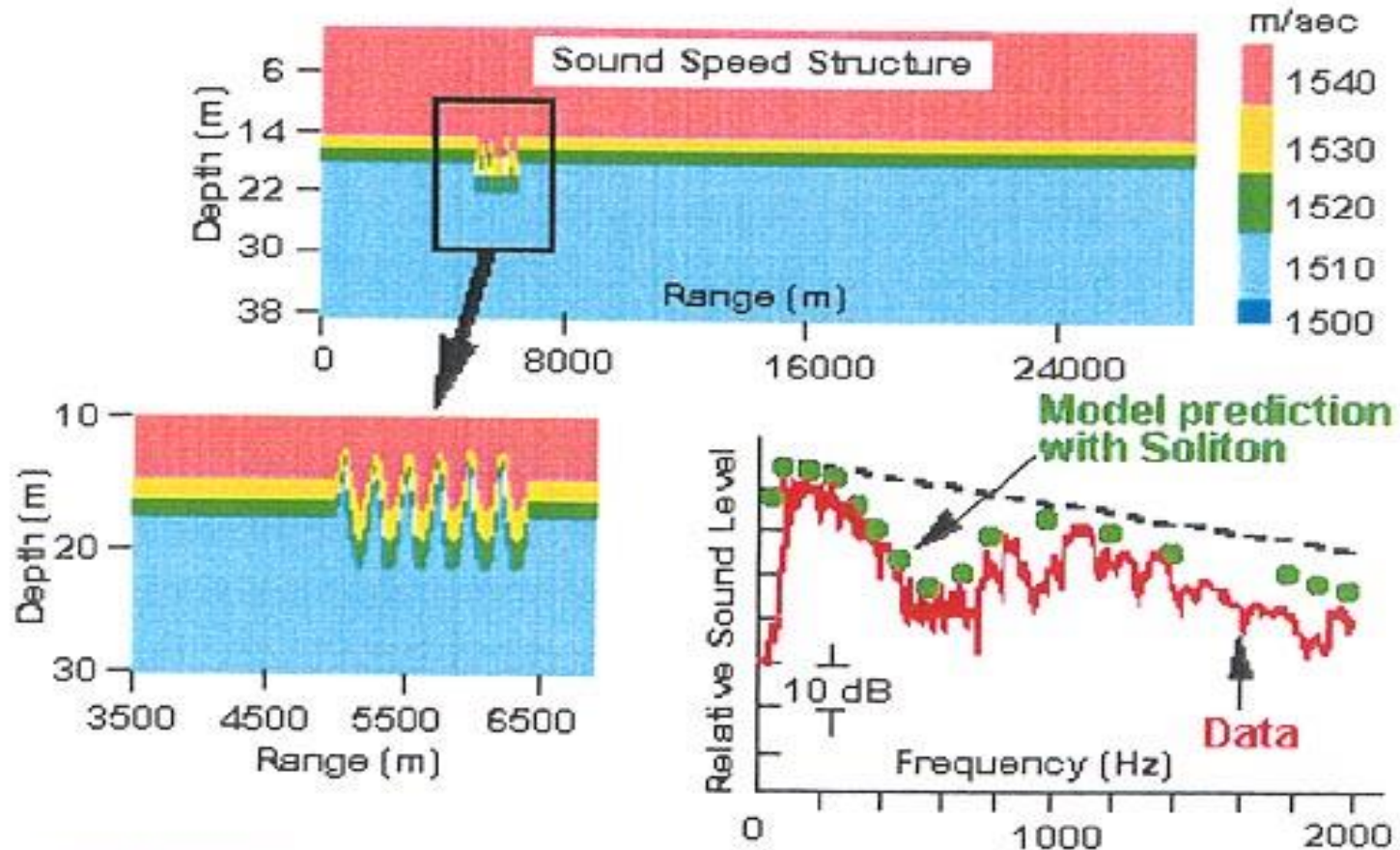
Courtesy of Norman Fraser, Photographer and Master Mariner,
Queensland, Aus.

DERA Turbulence probe Malin Shelf 1995



Acoustics

(ref: Zhou et al J Acoust Soc Am 90(4) 1991)



Green Circles: Acoustic model results using solitons in the environment

Requirements

⌘ Important therefore to predict:-

- Propagation of large amplitude internal waves
- Interaction with topography
- Internal wave-internal wave interaction
- Wave-wave interaction over varying topography

Mathematical formulation

Mathematical formulation

- ⌘ Governing equations
 - Numerical solution (CFD PHOENICS)

$$\frac{\partial \rho \phi}{\partial t} + \text{div}(\rho u \phi) = \text{div}(\Gamma_\phi \nabla \phi) + S_\phi$$

2 layer system:

- ⌘ Korteweg de Vries (KdV)

$$\frac{\partial \eta}{\partial t} + (c + \alpha_0 \eta) \frac{\partial \eta}{\partial x} + \beta \frac{\partial^3 \eta}{\partial x^3} = 0$$

- ⌘ Extended Korteweg de Vries (EKdV)

$$\frac{\partial \eta}{\partial t} + (c + \alpha_0 \eta - \alpha_1 \eta^2) \frac{\partial \eta}{\partial x} + \beta \frac{\partial^3 \eta}{\partial x^3} = 0$$

- ⌘ EKdV solitary wave solution
 - Michallet and Barthelemy JFM 366 1998

$$\eta(x, t) = a \frac{\sec h^2[\kappa(x - C_m t)]}{1 - \mu \tanh^2[\kappa(x - C_m t)]}$$

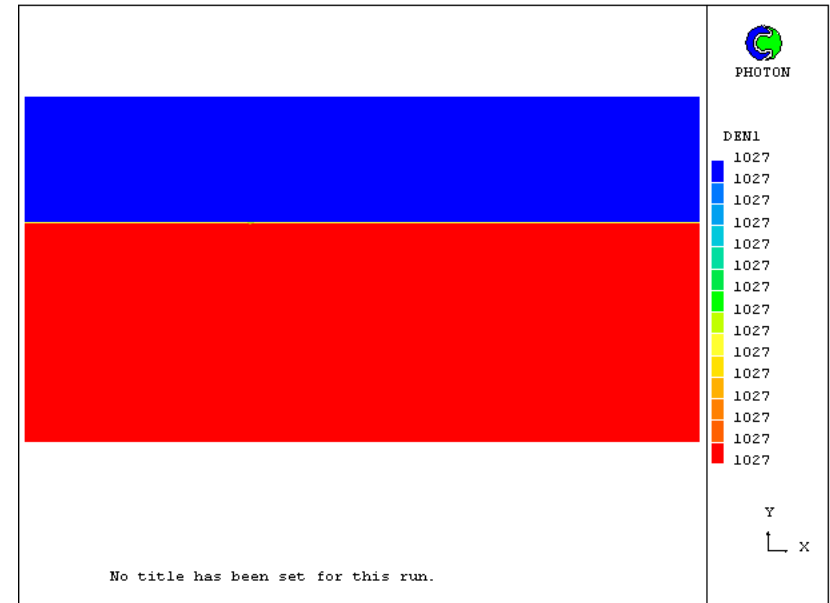
PHOENICS case studies

PHOENICS case studies

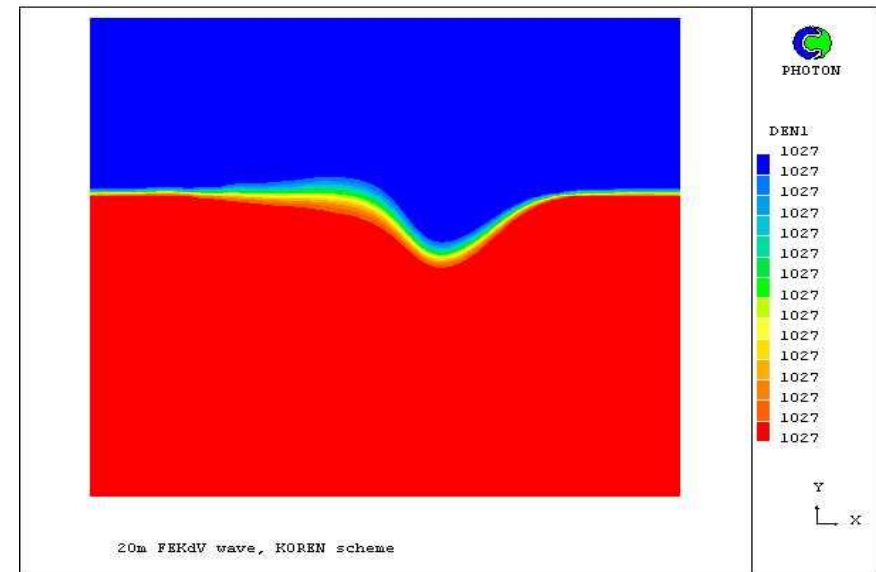
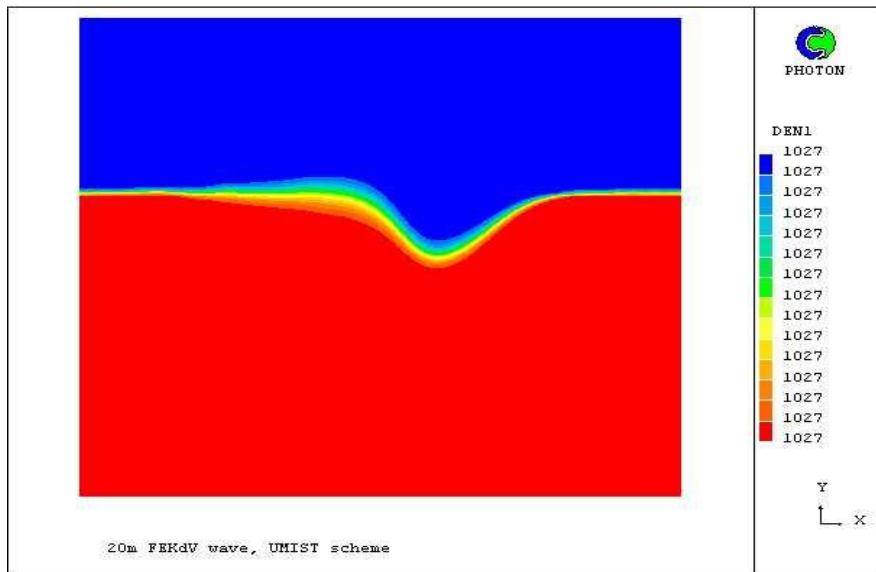
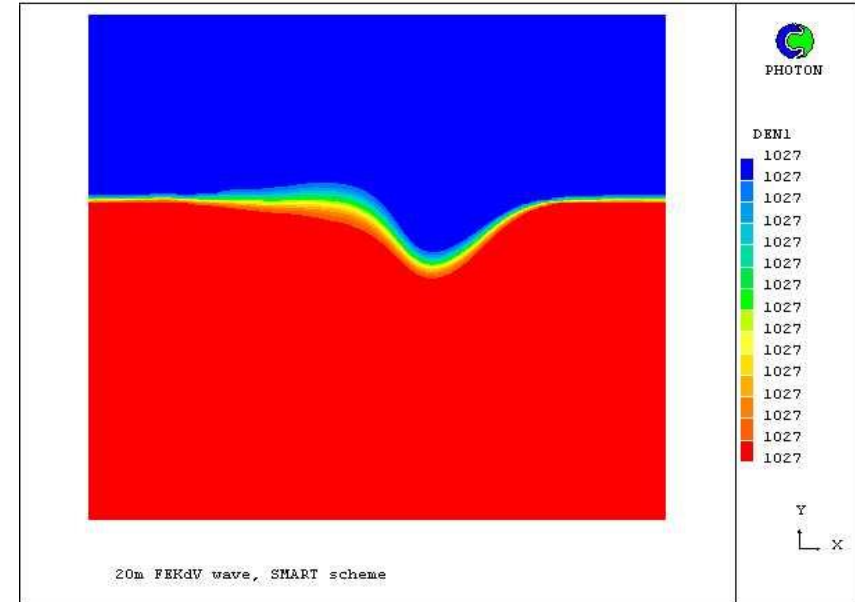
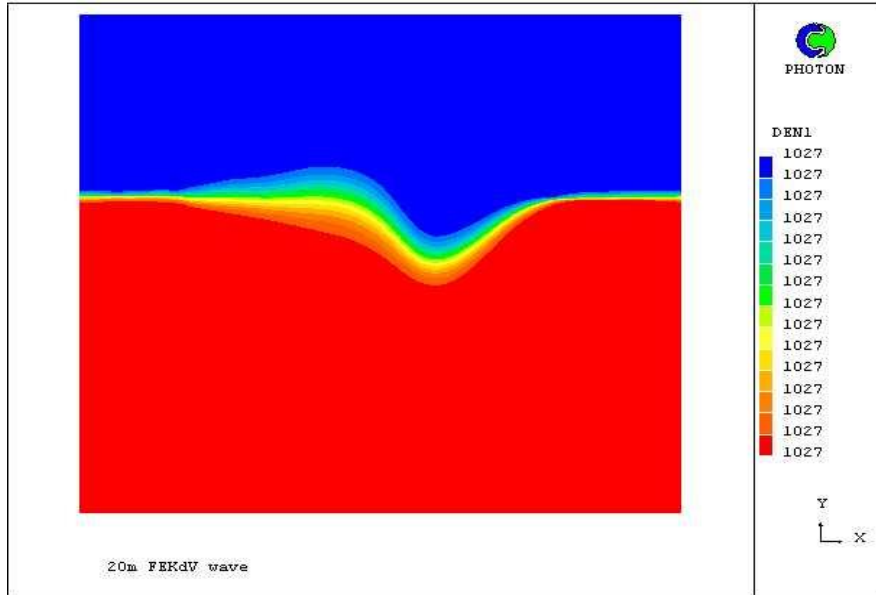
1. Propagation of small and large amplitude 2-D solitary waves
2. Interaction of colliding small and large amplitude 2-D internal waves
3. Propagation of small and large amplitude 2-D internal waves up a slope
4. Propagation of small and large amplitude 2-D internal waves up a slope and impingement on the slope
5. Propagation and interaction of 3-D large amplitude internal waves
6. Propagation and interaction of 3-D large amplitude internal waves over variable bathymetry

PHOENICS v3.2 Modelling

- ⌘ 2-D/3-D rectangular geometry
 - Blocked cells to represent topography
- ⌘ Staggered grid (uniform)
 - High order spatial upwind scheme
(dx~10m, dy~1m)
 - First order time discretisation (dt~20s)
Top layer~50m; bottom layer~90m
- ⌘ Rigid lid
 - No surface/bottom sources
- ⌘ Wave initialisation
 - Domain insertion
 - Via lateral boundary
- ⌘ Cyclic/fixed pressure/fixed flow boundaries



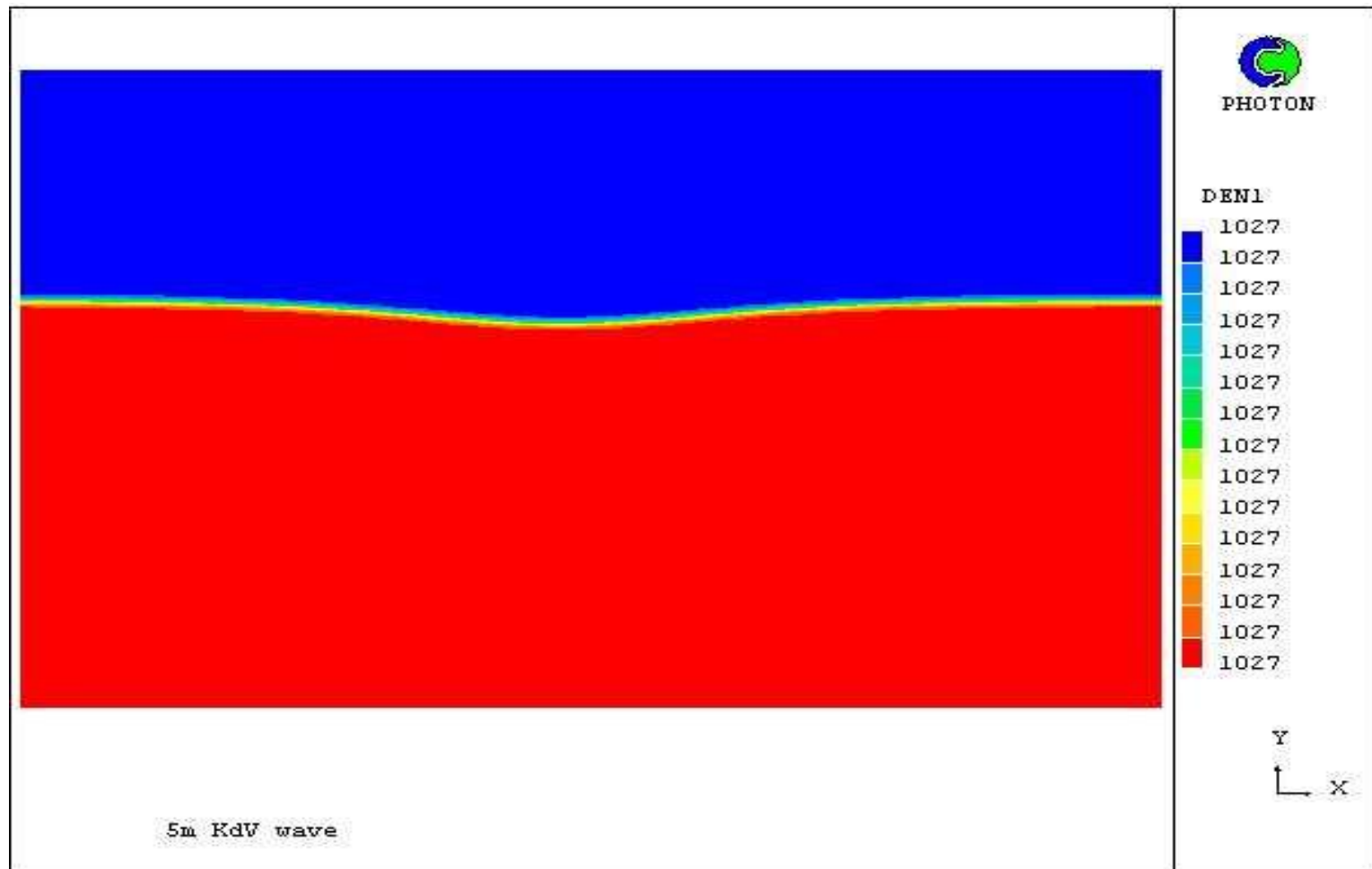
Differencing schemes



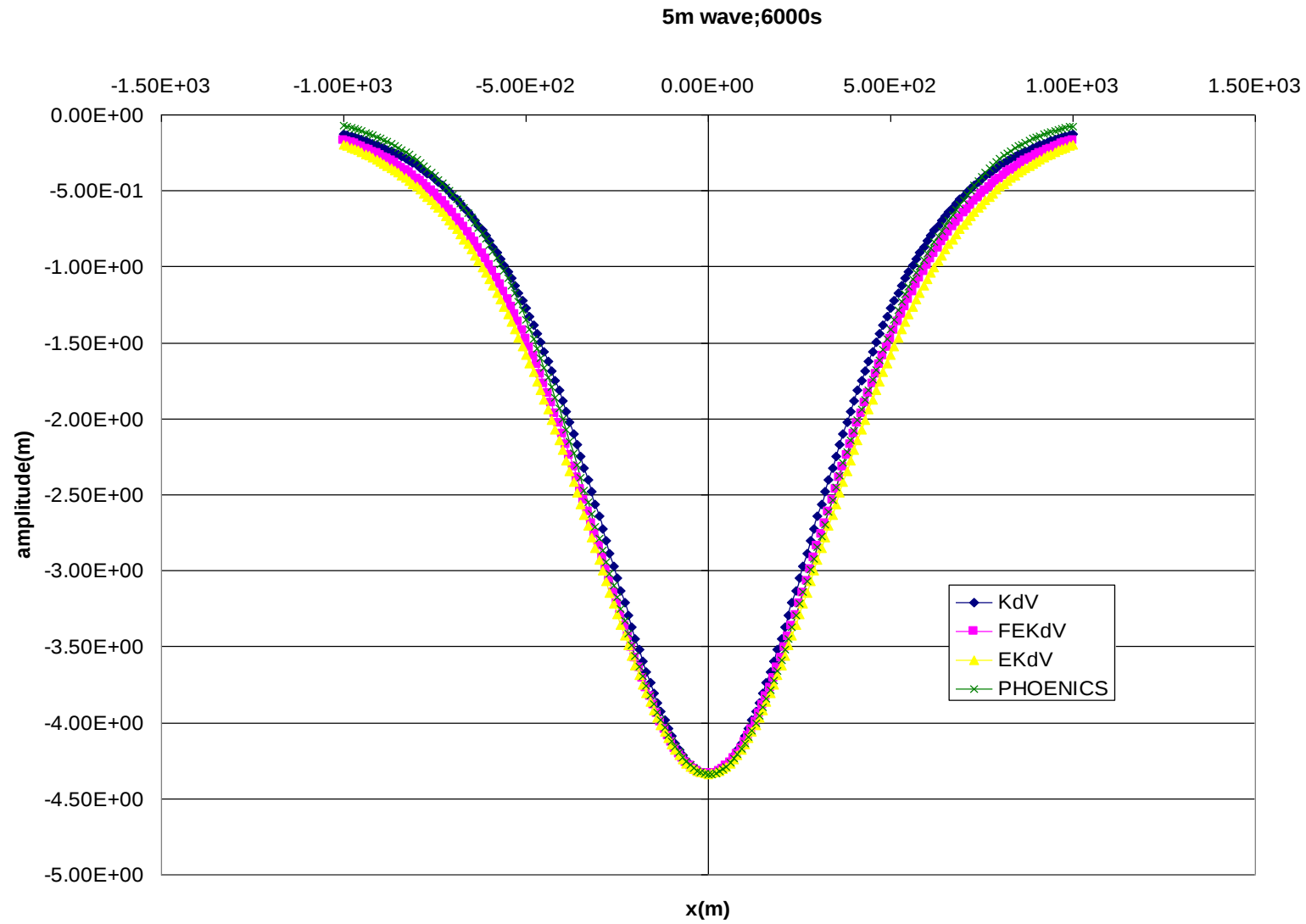
1. Propagation of solitary waves

- ⌘ 5m and 18m amplitude waves; 2 layer and continuous stratification
- ⌘ KdV should give good results for 5m wave, inaccurate for 18m wave
- ⌘ EKdV should give good results for both 5m and 18m waves
- ⌘ PHOENICS simulation
 - using cyclic boundary conditions (wave propagates in domain)
 - $dt=20s$; $dx=1m$; $dy=10m$
 - fixed flow at calculated wave phase speed on east boundary to 'freeze' wave; fixed hydrostatic pressure on west boundary
 - Effect of change in time step from 20s to 10s

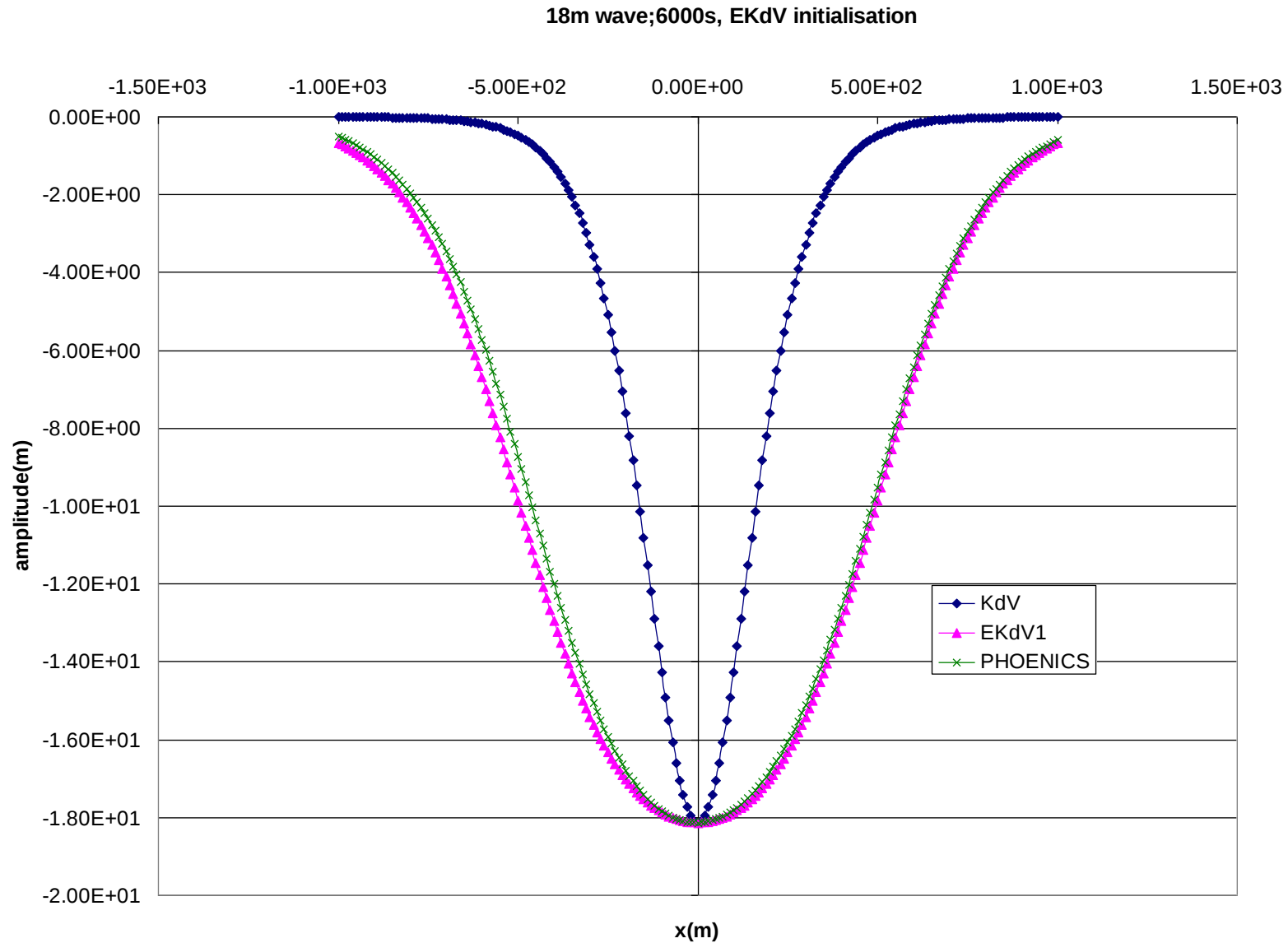
5m KdV wave (t=0)



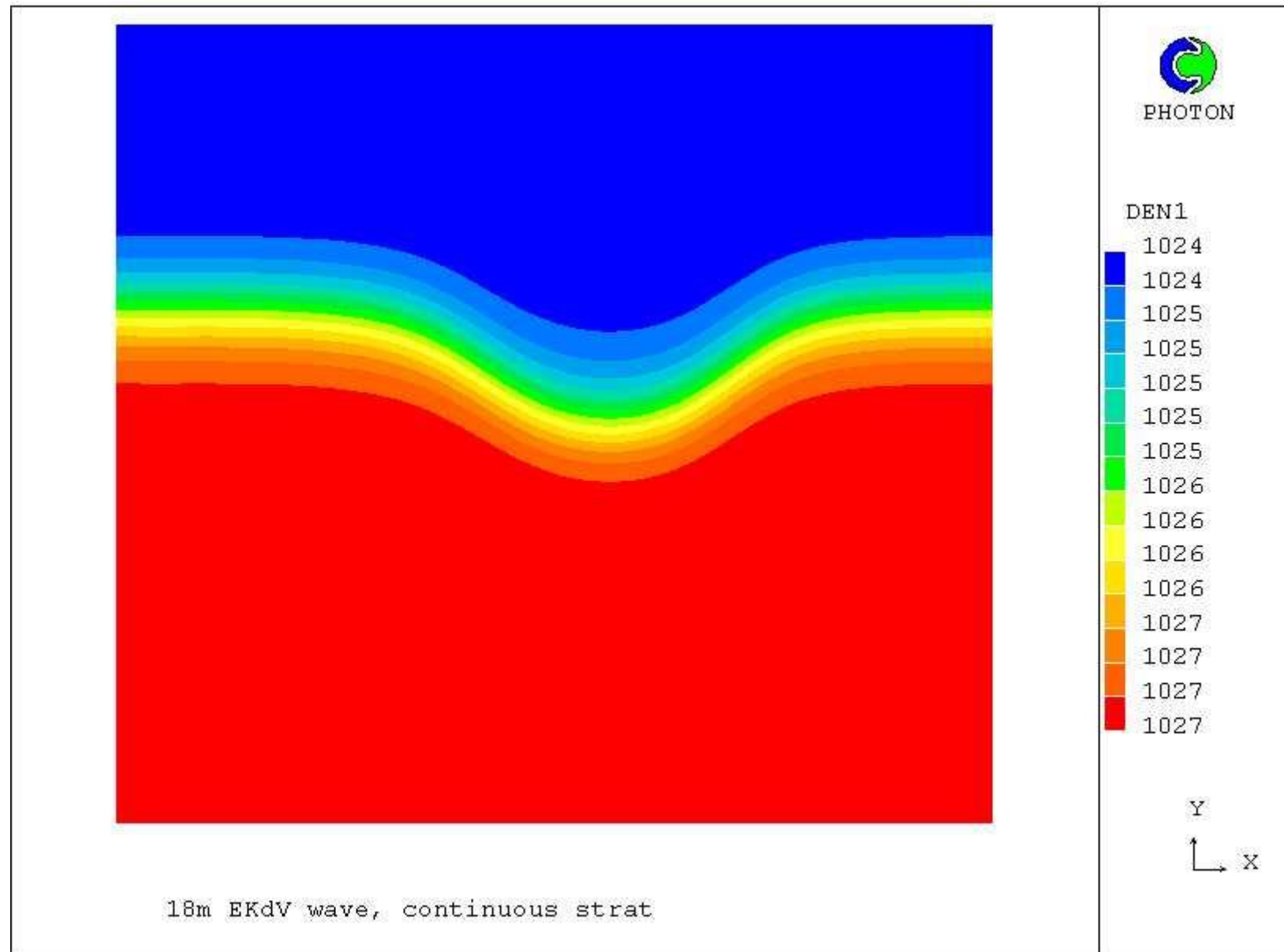
5m KdV wave (t=6000s)



18m EKdV wave (t=6000s,inflow=0.4m/s)

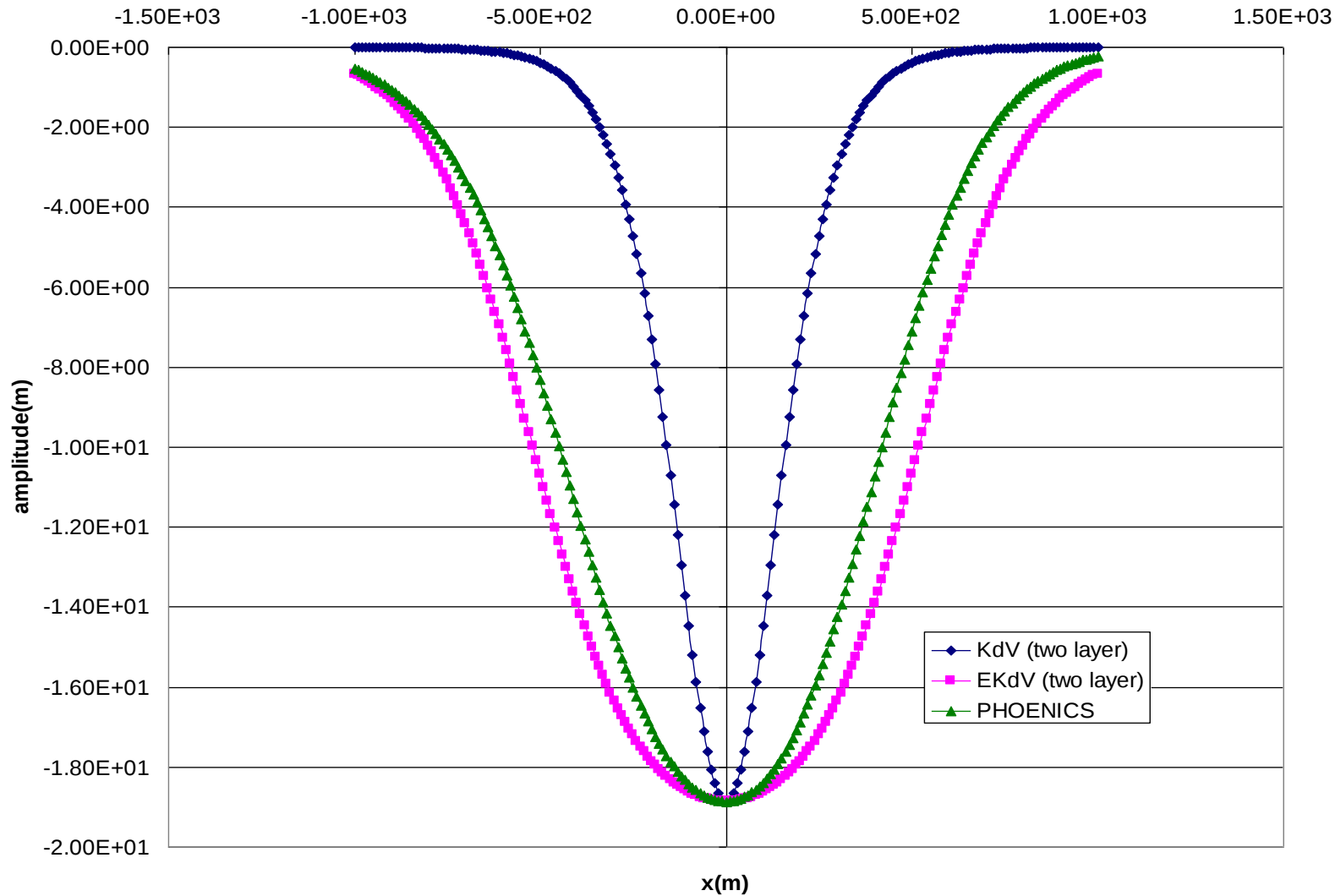


18m EKdV wave (t=6000s, inflow=0.9m/s, continuous stratification)



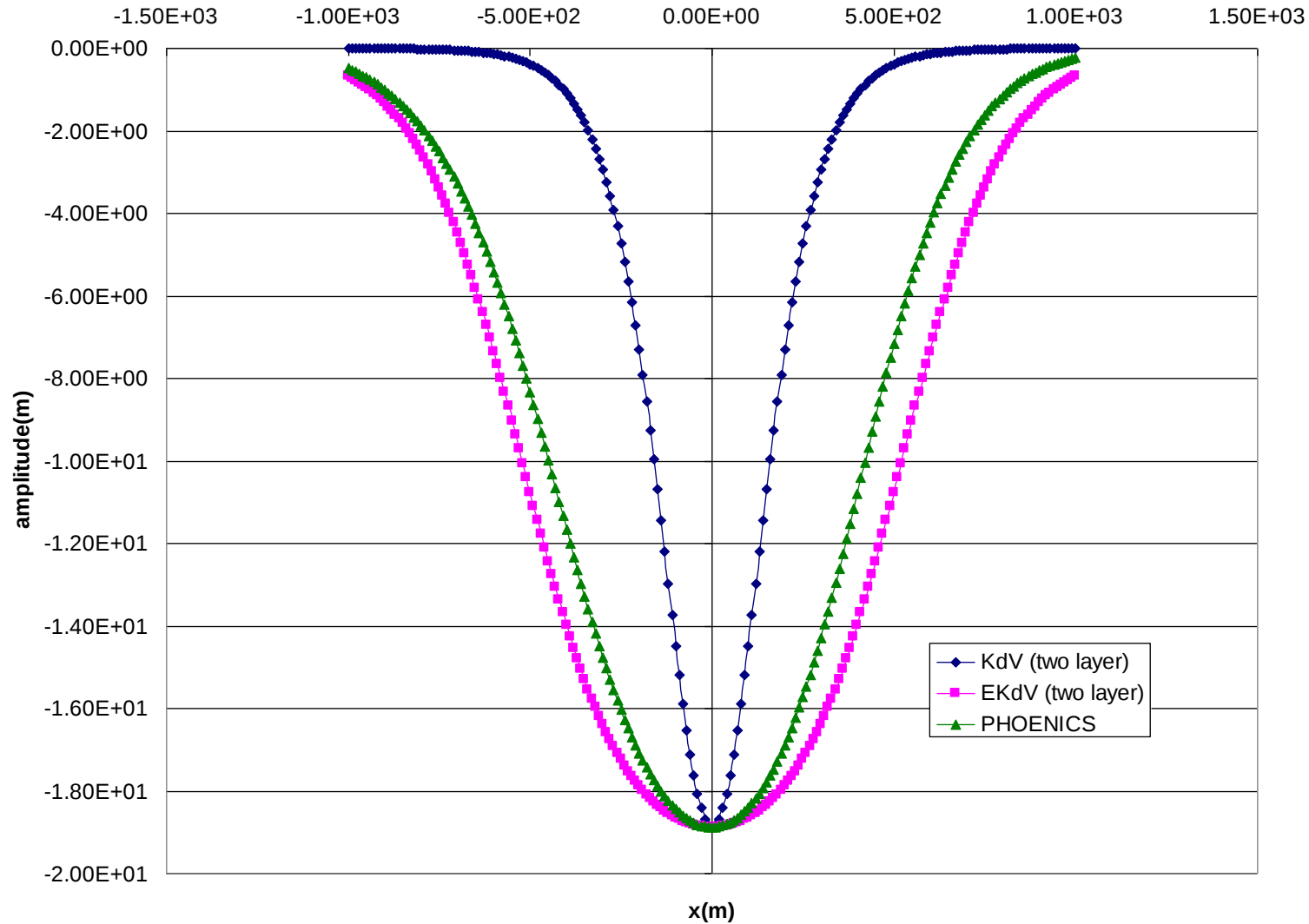
18m EKdV wave (t=6000s, dt=20s, inflow=0.9m/s, continuous stratification)

18m wave; EKdV initialisation; t=6000s; continuous stratification



18m EKdV wave (t=6000s,dt=10s, inflow=0.9m/s, continuous stratification)

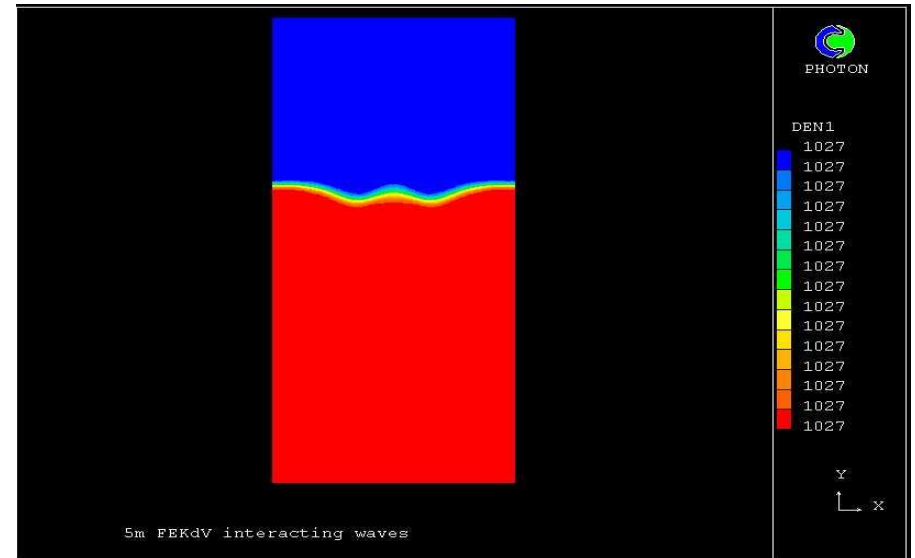
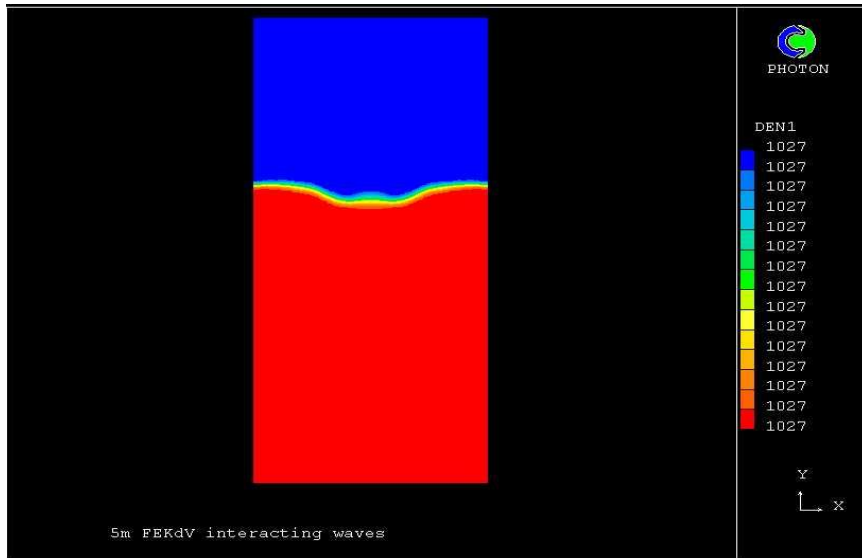
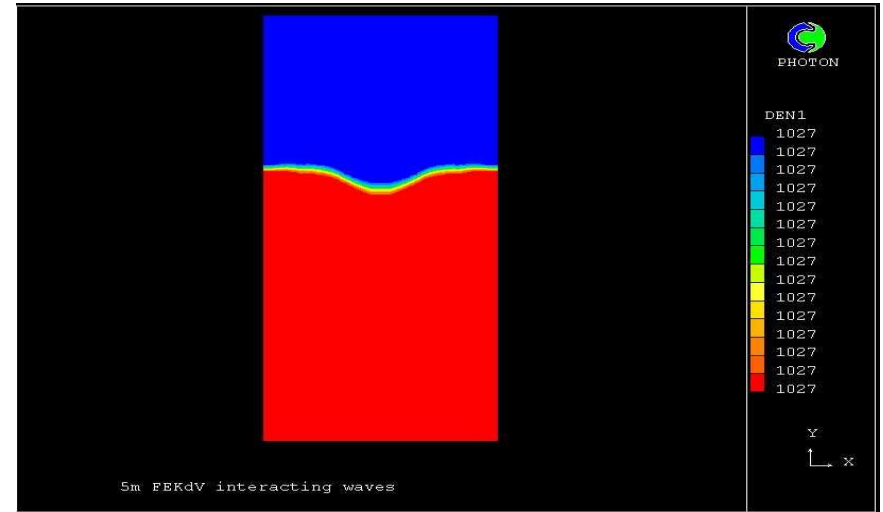
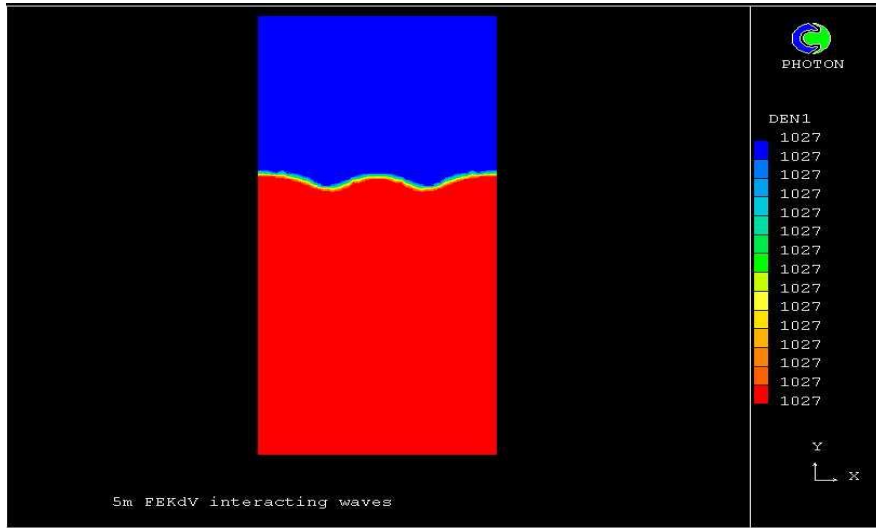
18m wave;EKdV initialisation;t=6000s,dt=10s;continuous stratification



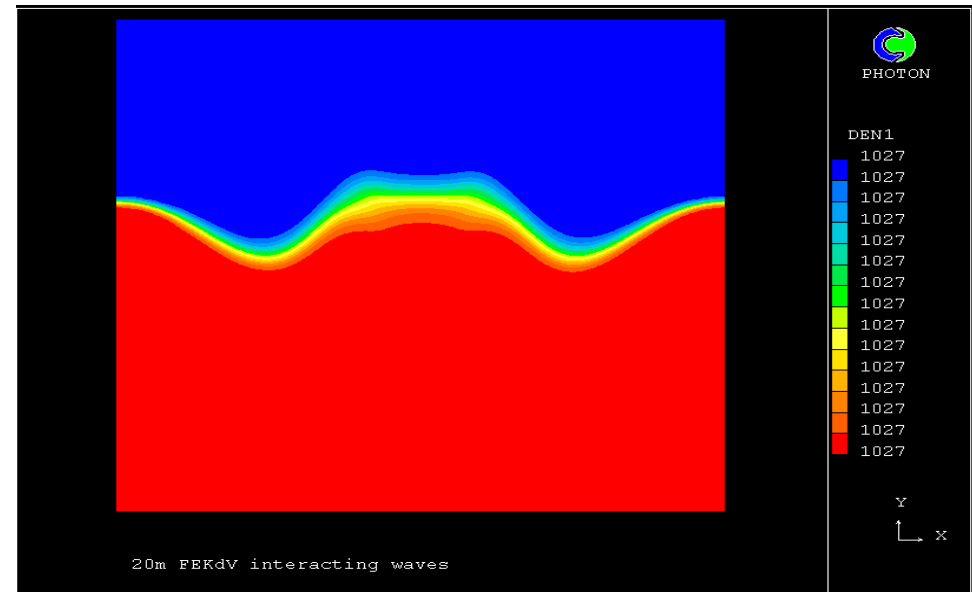
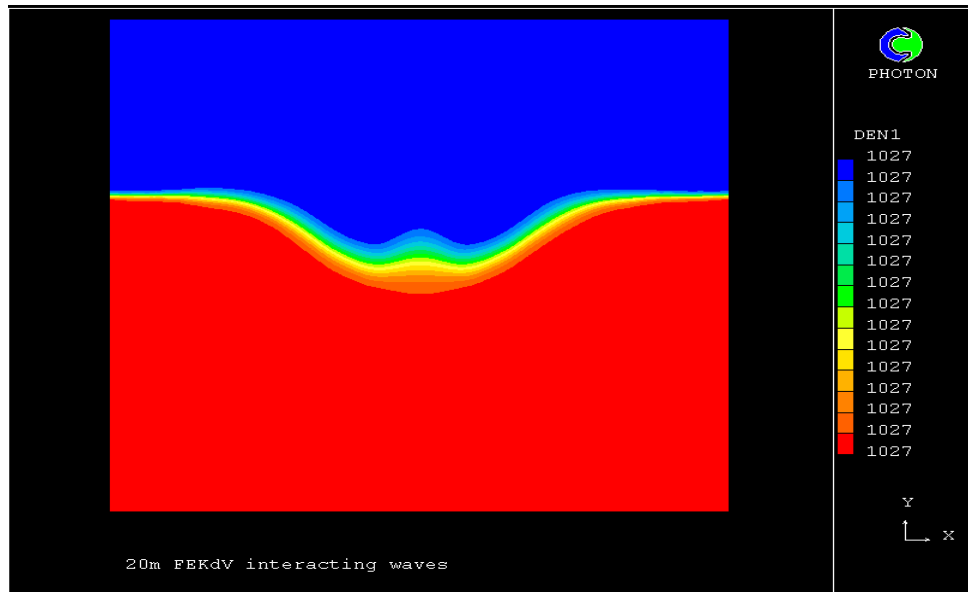
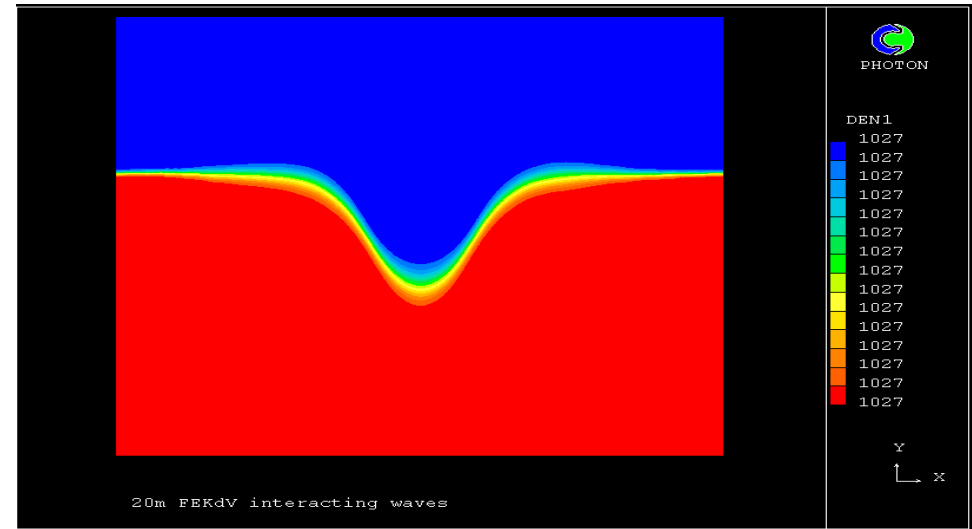
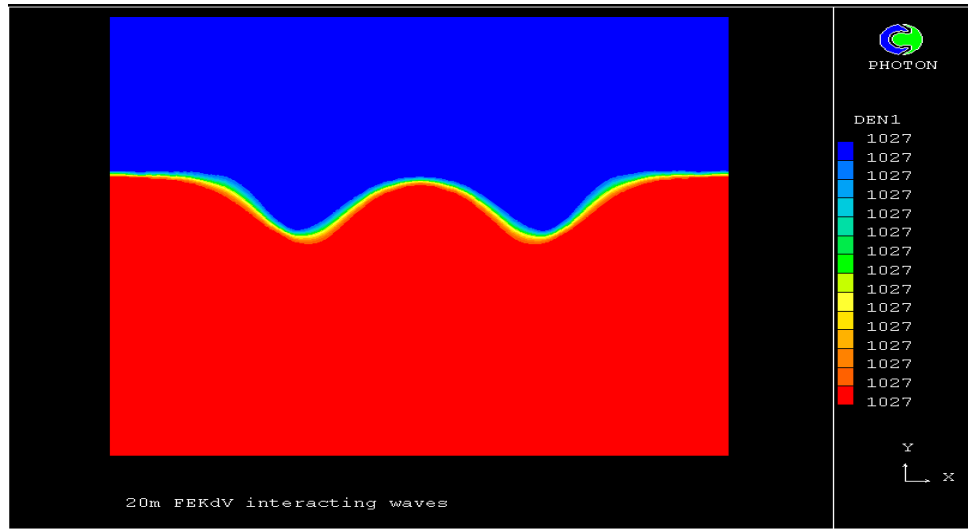
2. Colliding internal waves

- ⌘ 5m and 20m KdV and EKdV solitary waves
- ⌘ 2 layer environment
- ⌘ Water depth 140m
- ⌘ PHOENICS simulation
 - cyclic boundary conditions
 - $dt=20s$; $dx=1m$; $dy=10m$

5m interacting waves; 2 layer



20m interacting waves; 2 layer



3. Propagation of internal waves up a slope

- ⌘ 5m KdV and 20m EKdV solitary waves

 - 2 layer environment

- ⌘ 20m wave

 - continuous stratification

- ⌘ Water depth 140m

- ⌘ Slope gradient=0.05

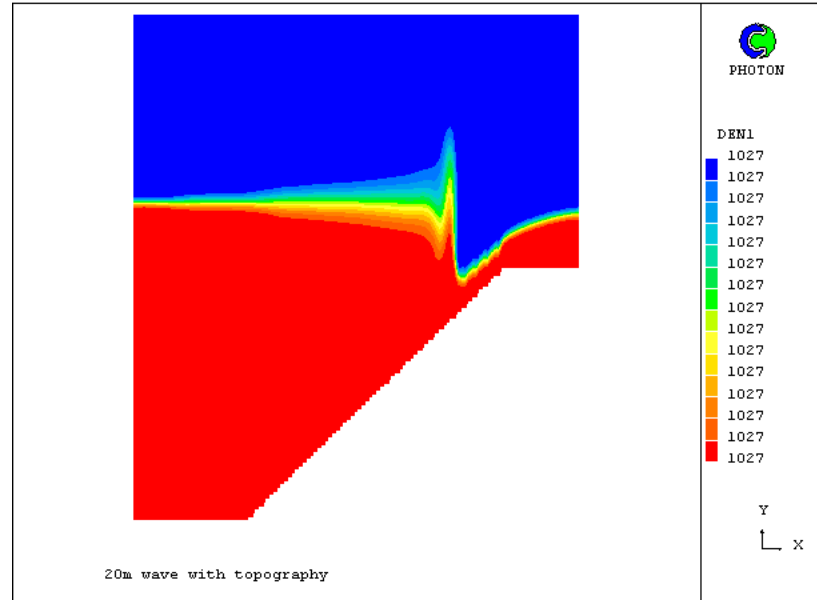
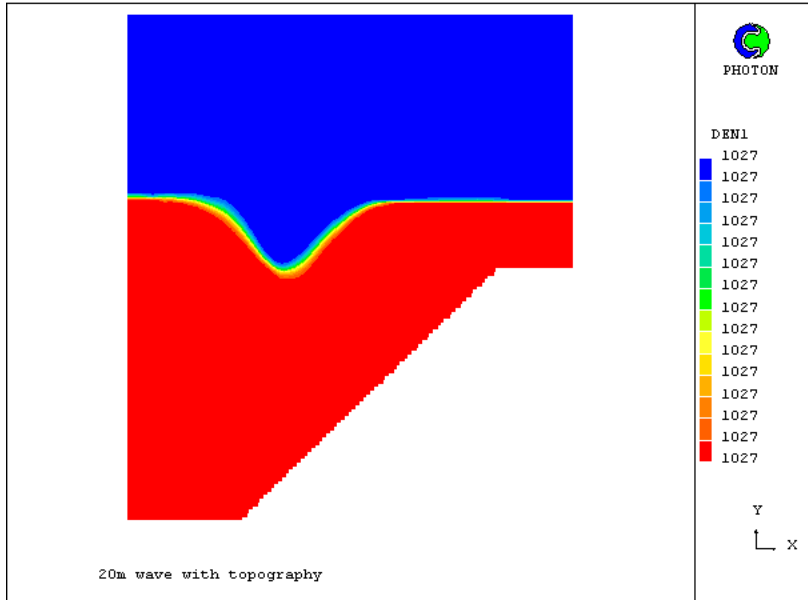
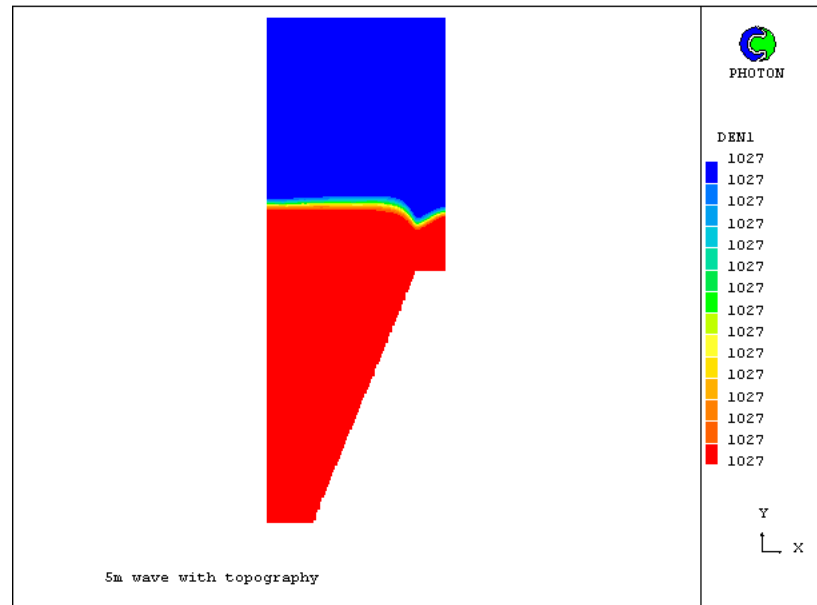
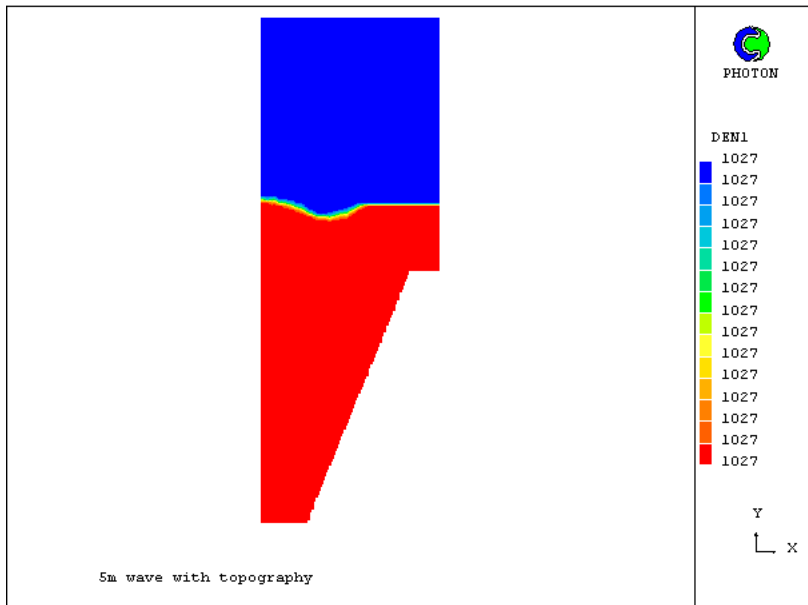
- ⌘ PHOENICS simulation

 - fixed pressure boundary conditions

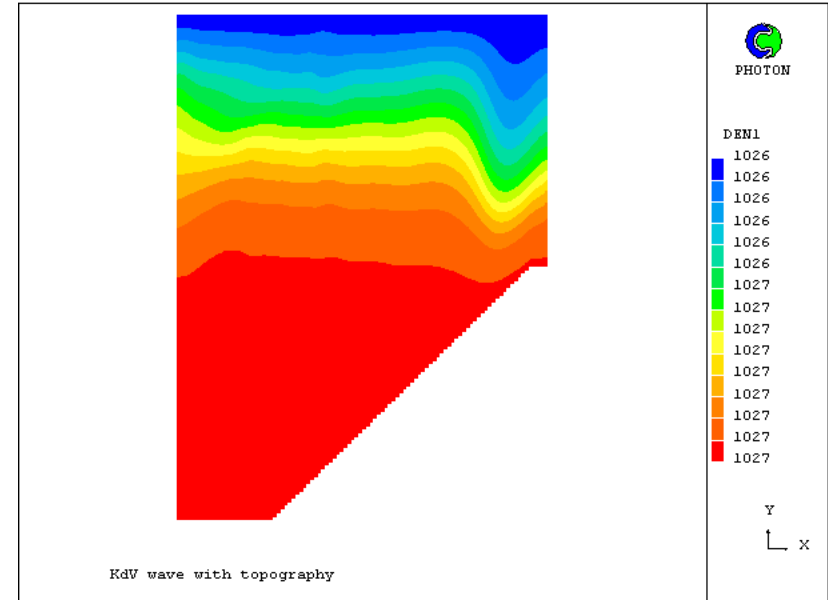
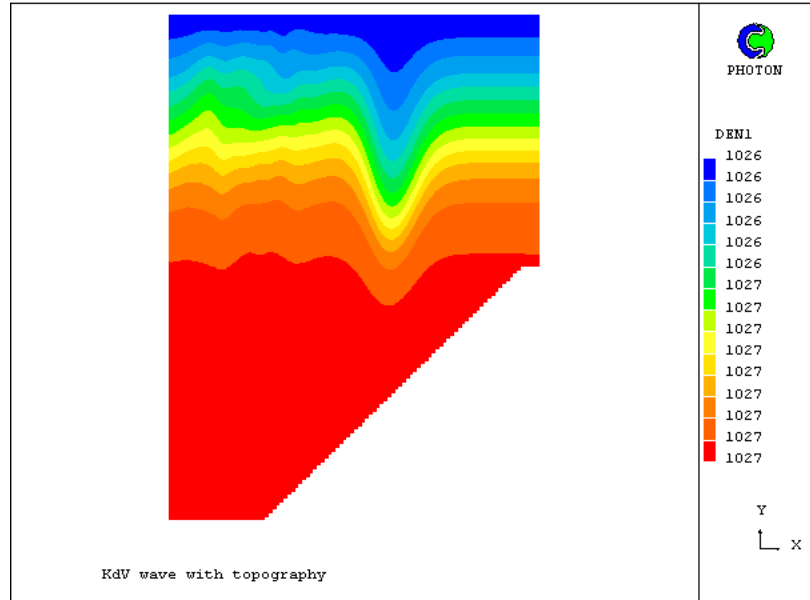
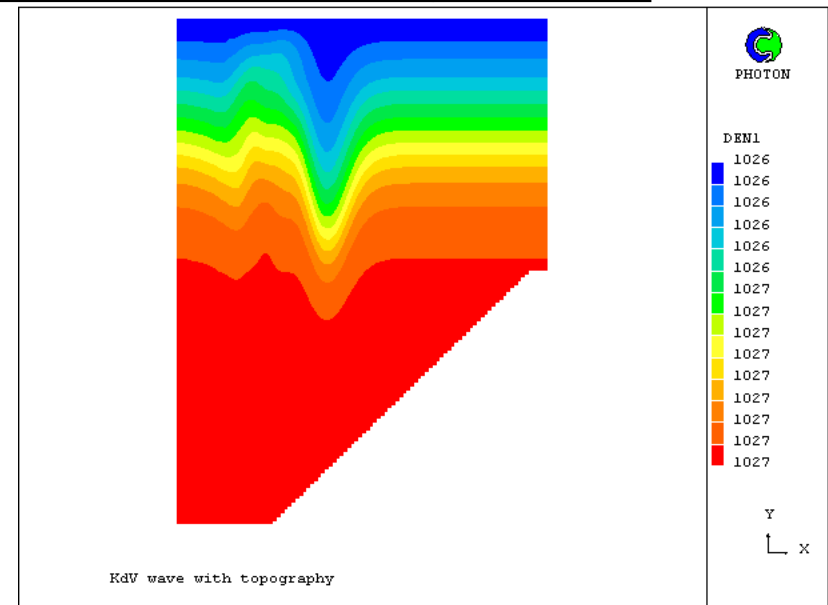
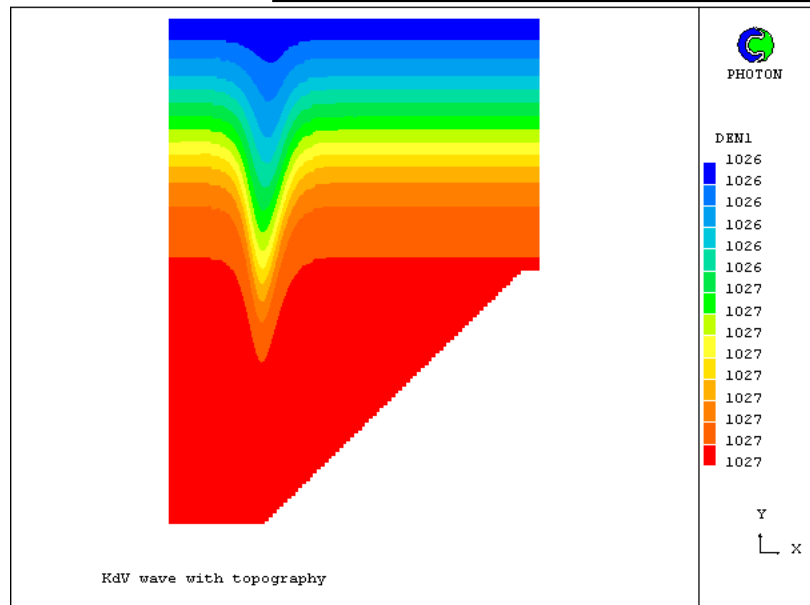
 - $dt=20s$; $dx=1m$; $dy=10m$

 - porosity used for slope blockage

5m/20m waves with topography; 2 layer



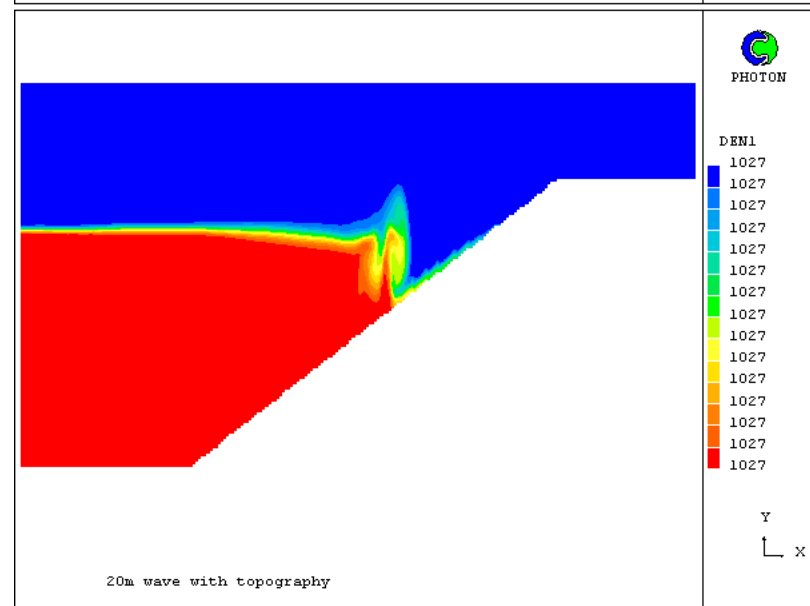
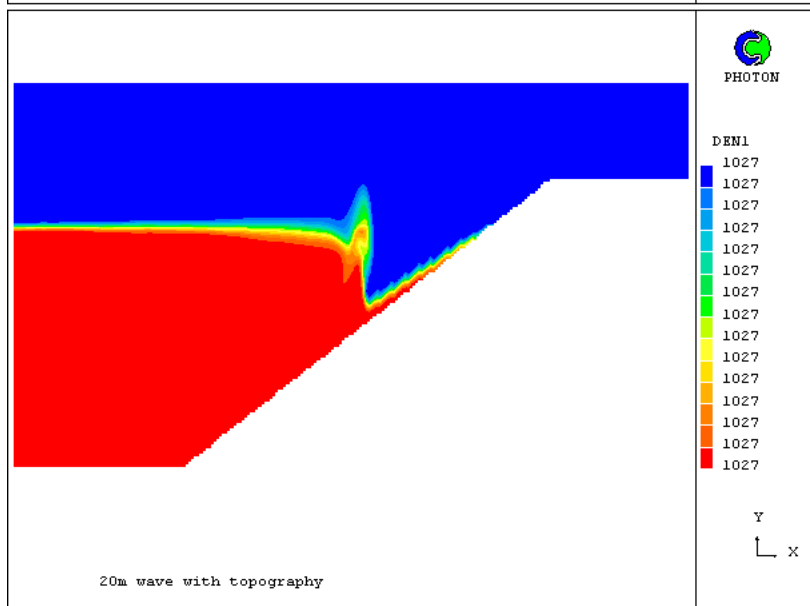
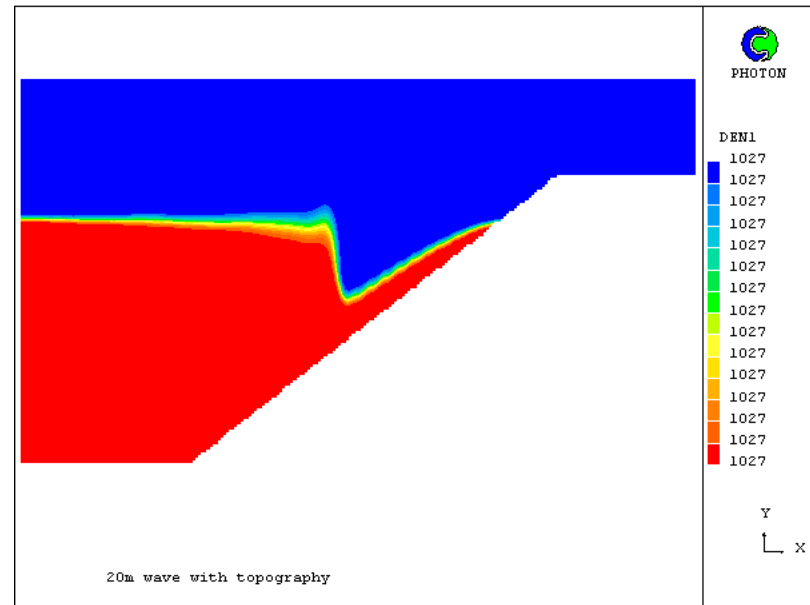
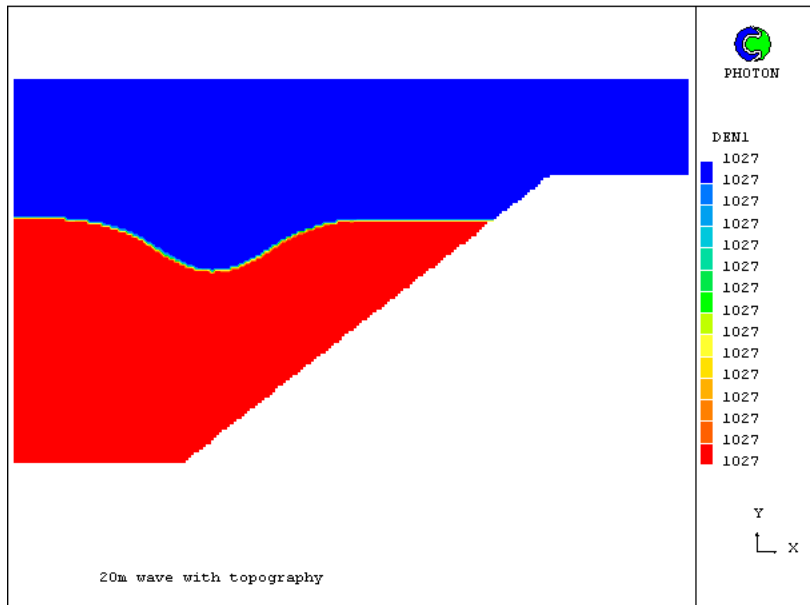
Continuous stratification/topography



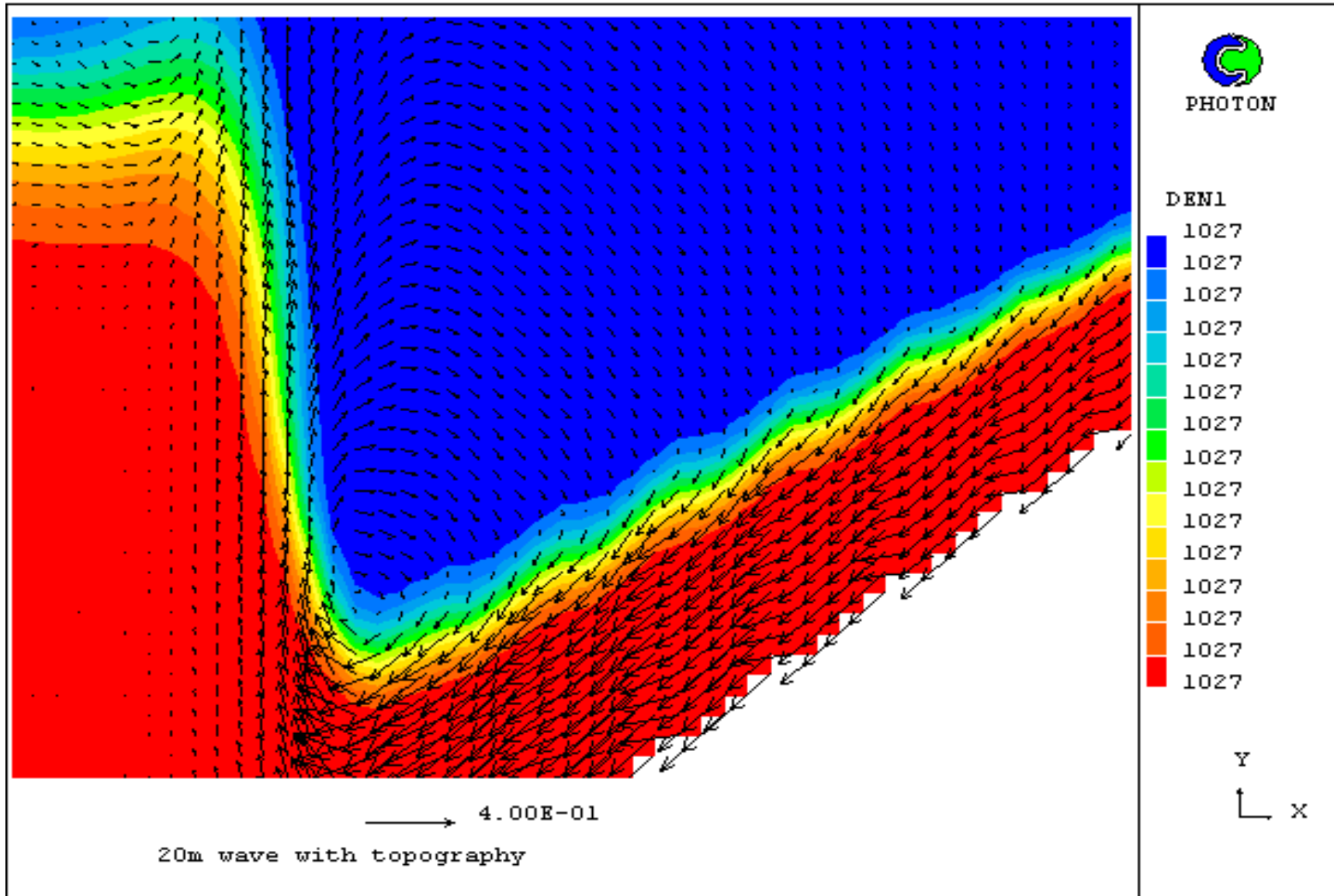
4. Impingement of internal waves on a slope

- ⌘ 20m EKdV solitary wave
- ⌘ 2 layer environment
- ⌘ Water depth 140m
- ⌘ Slope gradient=0.05
- ⌘ PHOENICS simulation
 - fixed pressure boundary conditions
 - $dt=20s$; $dx=1m$; $dy=10m$
 - porosity used for slope blockage

20m wave/topography interaction



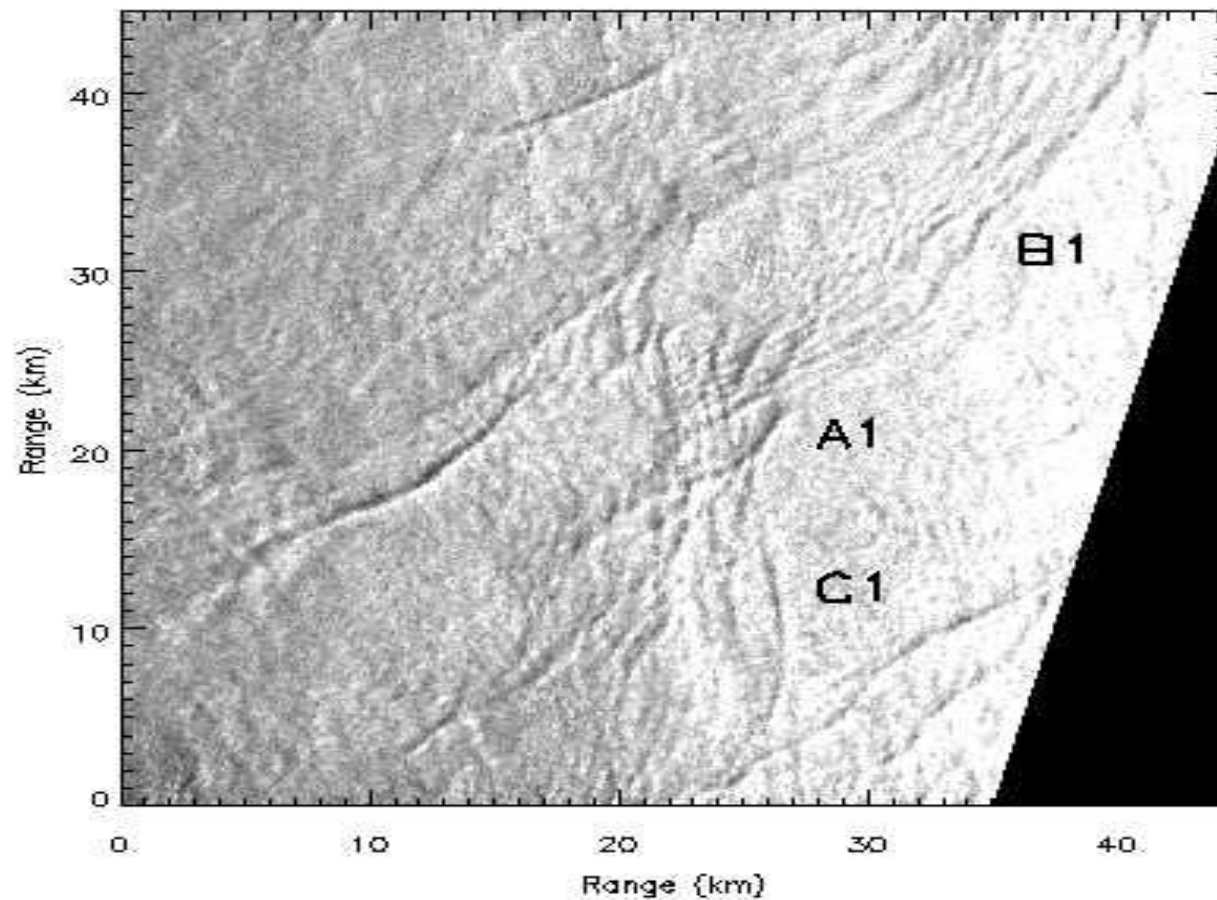
20m wave/topography interaction; velocity field



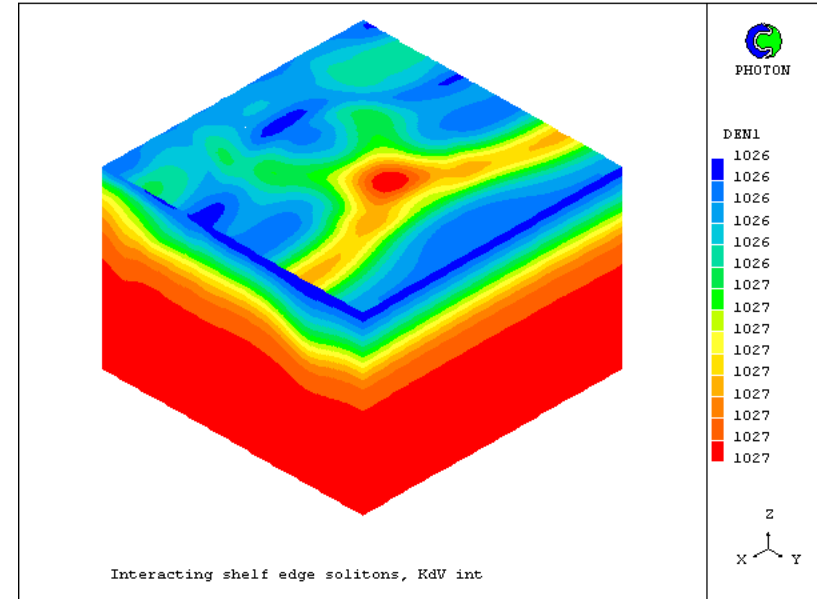
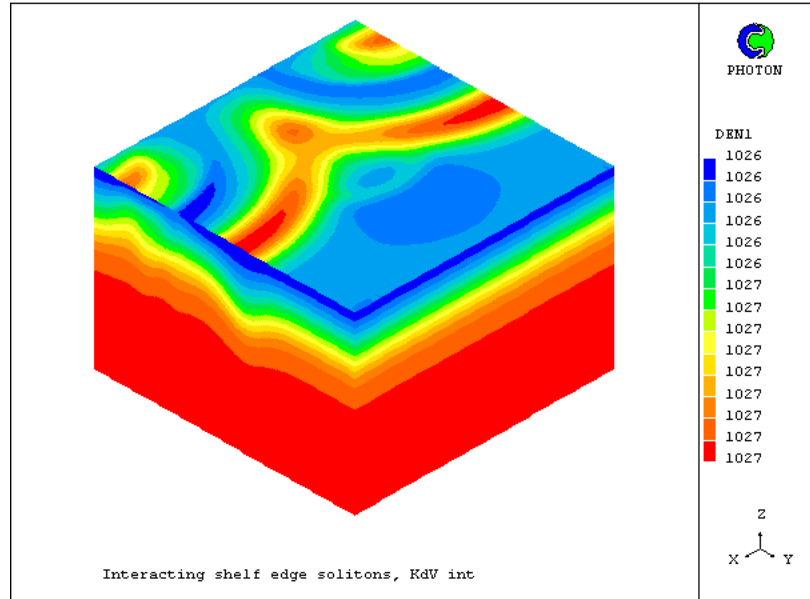
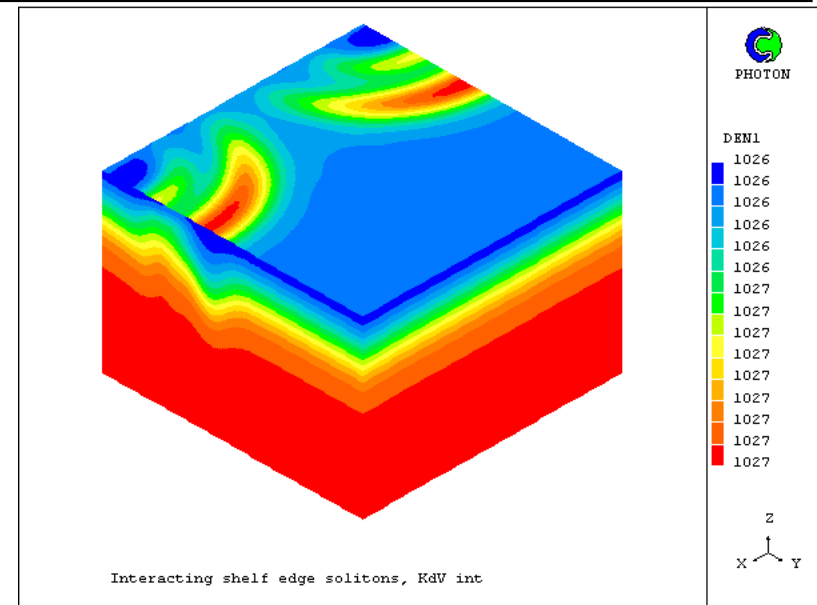
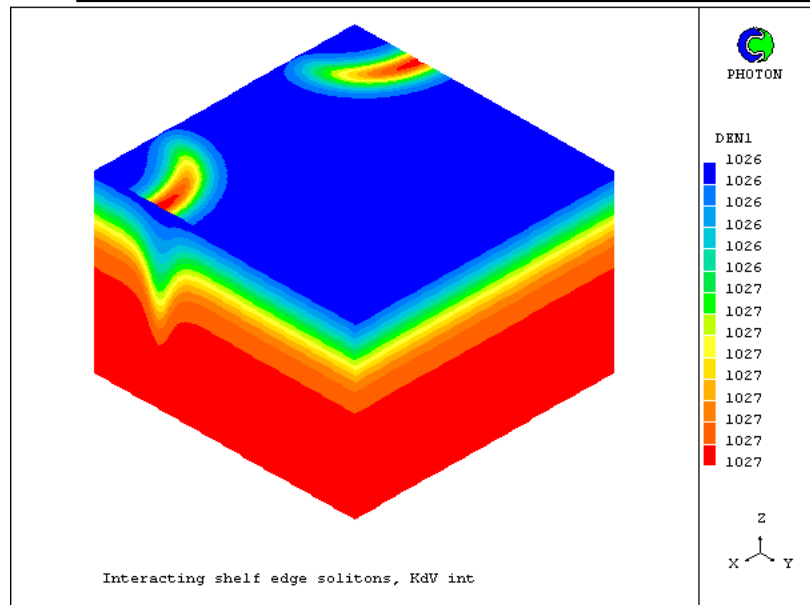
5. Interaction of large amplitude internal waves

- ⌘ Two 20m cylindrical waves travelling toward each other
- ⌘ Continuous stratification
- ⌘ Water depth 140m
- ⌘ PHOENICS simulation
 - solid free slip boundaries
 - $dt=20s$; $dx=5m$; $dy=40m$
 - domain sides contoured with density
 - domain top contoured with $v1$ velocity

ERS-1 SAR image, Malin Shelf, showing wave/wave interaction 1995



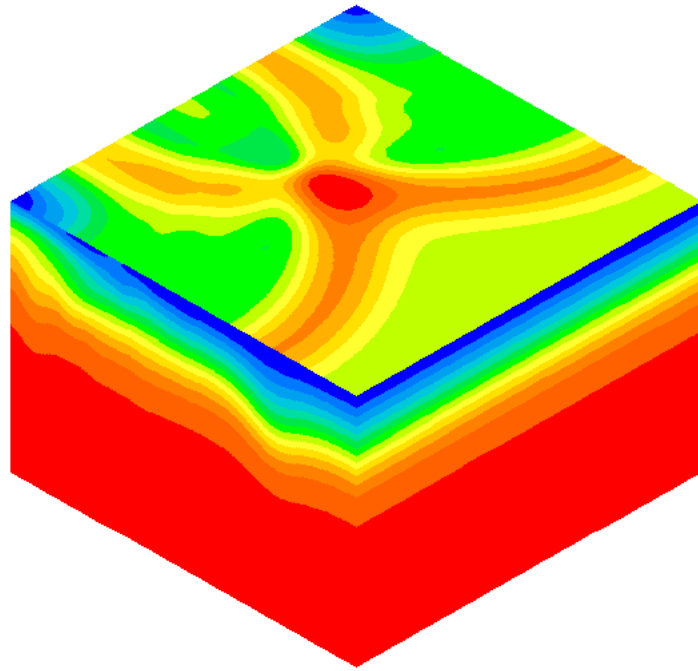
3-D interacting waves; continuous stratification



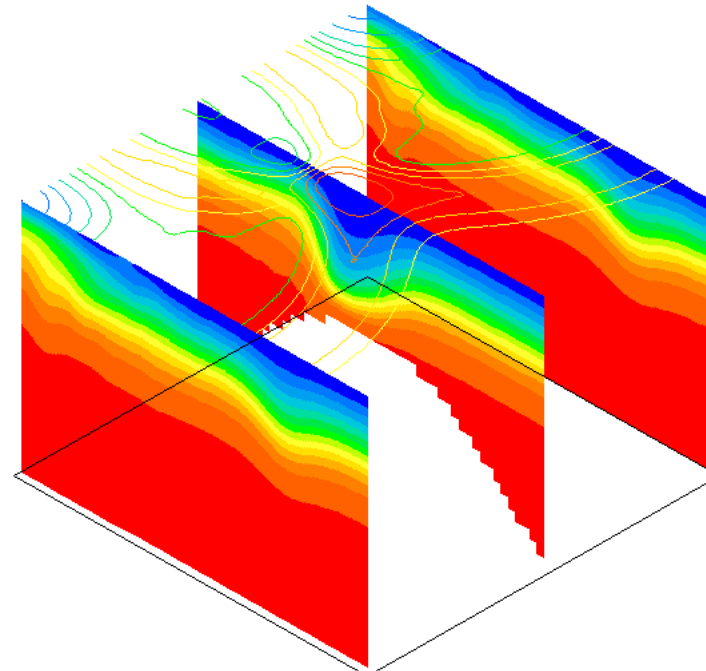
6. Interaction of large amplitude internal waves over variable bathymetry

- ⌘ Two 20m cylindrical waves travelling toward each other over seamount
- ⌘ Continuous stratification
- ⌘ Water depth 140m
- ⌘ PHOENICS simulation
 - solid free slip boundaries
 - $dt=20s$; $dx=5m$; $dy=40m$
 - domain sides contoured with density
 - domain top contoured with pressure
 - porosity used for seamount blockage

3-D interacting waves; continuous stratification; interaction with topography



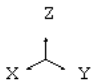
Interacting shelf edge solitons, KdV int



Interacting shelf edge solitons, KdV int



P1
-401
-385
-368
-351
-335
-318
-301
-285
-268
-251
-235
-218
-201
-185
-168



Conclusions

- ⌘ First stage assessment of PHOENICS code has shown that it has a good capability of simulating a wide variety of large amplitude internal wave flows
 - Good agreement has been obtained for solitary wave propagation
 - Physically plausible results obtained for other more complex flows

- ⌘ Future work will concentrate on:
 - More detailed comparison with available theory and experimental results
 - Use of higher order temporal scheme